

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVI.

APRIL 14, 1916.

No. 6

Professor R. A. Sampson, M.A., D.Sc., F.R.S., PRESIDENT, in the Chair.

Miss Francisca Herschel, Observatory House, Slough, Bucks ;
and
Arthur Henry Phayre, 15 Warrington Crescent, W.,
were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

John George Lincoln, Solicitor and Registrar of County Court, Lyndale, Ryden's Road, Walton-on-Thames, Surrey, and 222 Strand, W.C. (proposed by G. W. Nicolls); and
The Hon. Kathleen Robson, 26 Eaton Square, S.W. (proposed by Col. E. H. Hills).

The following were proposed by the Council as Associates of the Society :—

Dr. Henry Norris Russell, F.R.A.S., Princeton, New Jersey, U.S.A.; and
Professor Elis Strömgren, The Observatory, Copenhagen, Denmark.

Sixty presents were announced as having been received since the last meeting, including, amongst others :—

Specola Vaticana, Rome, Catalogo Astrografico, vol. ii., decl. +63°, calculate all' Osservatorio di Oxford, presented by the Vatican Observatory.

The Longitudes of Jupiter's Satellites, derived from Photographic Plates taken at the Cape Observatory in the Year 1913. By Professor W. de Sitter, Ph.D.

In agreement with the programme for future observations of the satellites of Jupiter drawn up by me at the request of the late Sir David Gill, and published in the *History of the Cape Observatory*, pages xcvi to c, plates were taken at the Royal Observatory, Cape of Good Hope, in the years 1913, 1914, and 1915. The plates taken in 1913 were sent to Leiden towards the end of that year. Those taken in 1914 and 1915 are still being kept at the Cape, but the times of exposures have been forwarded to me, so that the computation of tabular places could be taken in hand at once. The plates from 1913 have been measured here, and a concise account of the results derived from them is presented in this paper.

The aim of the investigation, of which these plates represent the first part, is to derive at chosen epochs the longitudes of the three inner satellites, in order to collect data for the determination of the libration and the inequalities of Group II., *i.e.* those with periods between 400 and 500 days. The longitudes must be determined from the co-ordinates x perpendicular to Jupiter's axis of rotation.

On each plate there are six exposures of Jupiter and its satellites, arranged in a line parallel to this axis, *i.e.* approximately perpendicular to the line along which the images of the four satellites produced by each exposure are projected on the plate. In consequence of this arrangement, the six images of any one satellite can be referred to the same division of the scale of the measuring instrument.

The co-ordinates x are only very slightly affected by the orientation of the plate, which need not therefore be very accurately known. In fact, an error of $30''$ in the orientation would even in extreme cases cause an error of less than 0.00004 in the co-ordinate of any satellite. The scale-value, of course, does affect the co-ordinates x . A change of scale is equivalent to a change in the adopted mass of Jupiter. Now this mass is known to within $1/35000$ of its amount, and consequently the scale to $1/100000$. We can thus eliminate the scale-value from the observed co-ordinates with great accuracy by introducing into the equations an additional unknown μ , of which the coefficient is $\frac{\partial x}{\partial \mu} = x$. Unless independent means exist to reduce all plates to the same scale, the value of μ must be supposed different for every plate. As a rule the value of x for the fourth satellite is the largest, so that, roughly speaking, the scale-value for the three inner satellites is derived from the fourth.

For the purpose of this reduction of all plates to the same scale, and for the determination of the orientation, two stars in

the neighbourhood of Jupiter were photographed on each plate, one exposure being taken before and one after the exposures of Jupiter, so that the mean hour-angle is nearly the same. The stars are C.P.D. -23° , 7316 and 7347. They have been observed on the meridian at the Cape on 1913 September 1 and 4. The distance and position-angle derived from these observations are

$$s = 63' \cdot 987 \qquad p = 89^{\circ} \ 0' \ 54''.$$

The plates were taken with the Victoria telescope of the Cape Observatory. The aperture had been cut down to 12 inches. The duration of exposure was for the satellites 10^s , and for the stars 90^s (sometimes 2 min.). The plates used are Ilford Monarch (backed), and the developer was Amidol restrained. The equatorial adjustment was determined on September 23 and 29. The mean of the two determinations was used for both series. The first series consists of 15 plates, extending from April 20 to May 25; the second of 16 plates, taken between August 19 and September 24.

The plates have been measured on the Repsold measuring machine of the observatory at Leiden. The first series was measured by myself, the second one by Mr. C. de Jong, an advanced student of astronomy in this university. The measuring apparatus need not be described here, it is entirely similar to other instruments of the same type. Five revolutions of the micrometer screw are approximately one millimetre, and as one minute is approximately two millimetres, one revolution is approximately equal to $0' \cdot 1$. The micrometer drum is read to thousandths of a revolution and the mean of the six images measured in two positions of the plate is computed to four places of decimals of a revolution, *i.e.* to $0' \cdot 00001$. The final residuals are of the order of some units of the fourth decimal of a minute of arc. The scale used with the instrument was made by the Société Genevoise. It is divided in millimetres, and its division-errors were determined at the International Bureau of Weights and Measures in Breteuil. They are very small, and in the central part, which is the only part used for the satellites, they run very regularly and do not deviate more than one- or two-tenths of a micron from a linear formula. Since the absolute value is of no importance to us, they can therefore be safely neglected. The average effect on the final place of a satellite is about $\pm 0' \cdot 00005$.

On the plate we take the projection of the declination circle through Jupiter, and a line perpendicular to it, as axes of η and ξ . The axes of measurement x and y are fixed to the microscope, the axis of y being parallel to the guiding cylinder. If Π be the angle between these two systems of axes, then the position-circle reads $\Pi + K$, K being the index-error, which must be determined from the observations of the standard stars. To effect this determination the plate is, by rotating the plate-holder, brought to such a position that the line joining the images of the two stars is nearly coincident with the axis y of measurement, *i.e.* nearly parallel to

the cylinder. The difference $A - B$ of the x co-ordinates is then measured by the micrometer. Let the position-circle reading be P_{AB} , and take $P'_* = P_{AB} + \frac{A - B}{s}$, A being the preceding star.

Then, if P_* be the apparent position-angle of the stars, we have $K_1 = P'_* - P_*$. A further correction K_2 is necessary, depending on the equatorial adjustment and on the distance on the plate from the middle point between the stars to the image of Jupiter. Then $K = K_1 + K_2$. The correction K_2 is very small, and seldom exceeds $10''$. It could very well have been neglected.

To measure the distance of the standard stars, the plate is then turned to the position-angle $P'_* + 90^\circ$, and the images of both stars are projected on the scale. From this distance, expressed in scale-divisions, we determine the value of one scale-division in minutes of arc. To reduce the revolutions of the screw to scale-divisions, one division of the scale $[10.0 - 10.1]$ is measured by the micrometer screw. Thus all measures can be reduced to nominal minutes of arc. These would be true minutes of arc if the adopted distance of the standard stars were exact, which most probably it is not. To reduce them to true they must be multiplied by a factor $1 + \mu$, which is derived from the measures of the satellites by introducing the constant μ as unknown into the equations of condition, as has been explained above. It should be mentioned that the distance of the standard stars was measured twice, once before and once after the measures of the satellites, so that any progressive change of scale-value due to heating of the instrument by the observer's body or to other causes should be eliminated in the mean. With very few exceptions, however, the results of the two measures agreed within the errors of pointing.

For the measurement of the satellites the plate is brought as nearly as possible to the position-angle $P'_0 = P + K$, P being the *apparent* position-angle (*i.e.* including refraction and aberration) of the projection on the celestial sphere of a line through the centre of Jupiter perpendicular to the adopted fundamental plane of the theory (which is the mean equator of Jupiter). If the position-circle reading be P_0 , then the measured co-ordinates x require a correction $\delta x = y \sin (P_0 - P'_0)$. It is not difficult to keep the differences $P_0 - P'_0$ within a few seconds, so that this correction can be neglected. The six images of each satellite are now arranged in a line parallel to the axis y of measurement. Two pointings are first made on the nearest scale-division, then one pointing on each of the six images, and then again two on the scale. During all these pointings the microscope is clamped to its bridge. In the Leiden instrument the whole bridge with the microscope clamped to it is rotated about a horizontal axis to change from plate to scale and inversely, by which device the "error of projection" has been got rid of, as it is necessarily the same for all positions of the microscope. After the measures of one satellite are thus completed, the microscope is unclamped and moved to the next

satellite. The measures have always been made in the same regular order. If the images be denoted by a, b, c, d, e, f , then one half of the measures is made in the order $a, b, \dots f$ and the other half in the reversed order. After rotating the plate through 180° the same measures are repeated, but the satellites which were first measured in the order $a, b, \dots f$ are now measured in the order $f, e, \dots a$, and inversely.

The image of the planet itself has not been measured. It has repeatedly been pointed out that this would introduce grave systematic errors [see, *e.g.*, *Gron. Publ.*, xvii. pp. 50-52; "Elements and Masses," *Proceedings, Amsterdam*, vol. x. pp. 660-661]. The observations thus give only the *differences* of the x co-ordinates of the satellites. To get a more symmetrical form of equations I have used the differences of each satellite from the mean of the four.

Let the mean error of one pointing on one image of a satellite be ϵ . We denote the six images of the four satellites by $a_1, b_1, \dots f_1, a_2, \dots$ etc., and the mean of the microscope readings on the six images of each satellite is called m_1, m_2 , etc., thus $m_1 = \frac{1}{6}(a_1 + b_1 + \dots f_1)$. The square of the mean error of one difference $a_1 - m_1, b_1 - m_1, \dots a_2 - m_2, \dots$ etc., is then $\frac{5}{6}\epsilon^2$. The same difference derived from the measures in position II. of the plate has the same true value and the same mean error. Thus the difference between these two differences is composed exclusively of accidental errors of pointing, and is independent of the errors in the position of the image on the plate. The square of the mean value of this difference is thus $\frac{5}{3}\epsilon^2$. In this way the error ϵ has been derived for every plate. Let, further, σ be the mean error of the mean of two pointings on the scale, then the square of the mean error, of the projection of the satellite on the scale (mean of six images), *i.e.* of one co-ordinate x determined from the measures in one position of the plate, is $\frac{1}{6}\epsilon^2 + \frac{1}{2}\sigma^2$. Now put $x_0 = \frac{1}{4}(x_1 + x_2 + x_3 + x_4)$ and $x_i = x_i - x_0$, then the square of the mean error of x_i is $\frac{1}{8}\epsilon^2 + \frac{3}{8}\sigma^2$, and the difference of the same x_i as determined from the measures in the two positions of the plates has a mean error whose square is

$$\zeta^2 = \frac{1}{4}\epsilon^2 + \frac{3}{4}\sigma^2.$$

The error σ can easily be derived from the pointings on the scale. Combining this with the value of ϵ found as above, we can compute the value of ζ . On the other hand, ζ can be derived directly from the comparison of the values of $x_i = x_i - x_0$ as derived from the measures in the two positions.

Of the first series, which was measured by myself, some plates have, for various reasons, been measured twice. On the plates of the second series Mr. de Jong made two complete sets of measures of the satellites on every plate. By comparing these two, and the two measures of the plates measured twice by myself, we get a third determination of the error ζ .

In carrying out these determinations of errors I have used average errors instead of mean ones. The results, expressed in thousandths of a revolution, *i.e.* in units of $\frac{1}{5}$ of a micron, are given below.

	de Sitter.	de Jong.
ϵ	± 12.0	± 10.0
σ	2.4	2.0
ζ from ϵ and σ	6.3	5.3
ζ from Pos. I.-II.	7.5	7.5
ζ from plates measured twice	6.4	6.3

For comparison I may add that from the Cape plates of 1903 taken with the astrographic telescope and measured by me in January 1905 on the measuring apparatus at Groningen, I found

$$\epsilon = \pm 8.2.$$

The images on the plates of 1903 were on the average decidedly better than on those of 1913. This may explain a part of the increase of my value of ϵ . Part of it must probably be ascribed to the deterioration of my eyesight with increasing age. Perhaps also the circumstance that the speed of the screw at Groningen was only half of that at Leiden (a thousandth of a revolution at Groningen being $\frac{1}{10}$ micron) may have some influence. It is rather remarkable that, notwithstanding the smaller values of ϵ and σ for Mr. de Jong, still the directly derived values of ζ are the same for both observers.

To facilitate the comparison of ϵ and ζ for different plates, the value of ϵ for every plate was divided by the mean, so that the mean becomes 1.0, and the same was done with the two values of ζ . The plates were then arranged in order of increasing ϵ and the mean was taken of every four successive plates. The average residuals η , derived from the final solutions, were treated in the same way. We thus find from the second series:

Mean ϵ .	ζ from Pos. I.-II.	ζ from Two Measures.	η .
0.74	0.75	1.12	1.14
0.92	0.96	0.74	1.00
1.04	1.21	0.74	0.91
1.29	1.08	1.38	0.95

The value of ζ determined in the first way is evidently correlated with ϵ , as could have been expected. Between ϵ and the second determination of ζ there does not appear to be any marked correlation, and between η and any of the other errors there is evidently no correlation at all. The policy which has been adopted, of assigning equal weights to all plates, is thereby justified.

The measures have been compared with my theory of the

satellites of 1908 published in "Elements and Masses" (*Proceedings of the Academy of Sciences at Amsterdam*, vol. x. pp. 653-673 and 710-729). The longitudes, radii-vectores, and latitudes have been computed entirely according to the formulas there given, with the only exception of the mean longitude of Satellite IV., to which a correction of $+0^{\circ}.0050$ was applied [see *M.N.*, vol. lxxi. p. 598]. Further, the libration was *not* included in the tabular places. The co-ordinates x and y are then determined by the formulas

$$\Delta' = \Delta + a\rho \cos B \cos (v - L'),$$

$$x = \frac{a'\rho}{\Delta'} [\sin (v - L') - \sin \beta \tan \frac{1}{2}i \cos (L' - \Omega)],$$

$$y = \frac{a'\rho}{\Delta'} [\sin \beta \cos B + \sin B \cos (v - L')].$$

Here a is the semi-major axis, and $a' = a \sin i'$. Further, the jovicentric longitudes, latitudes, and distances are :

of the satellite : $v, \beta, a\rho$,

of the earth : $L + 180^{\circ}, B, \Delta$.

The values of $v, \rho, i, \Omega, \beta = \sin i \sin (v - \Omega)$ must be computed for the time $t = t_0 - \tau$, where t_0 is the Greenwich mean time of the observation and τ is the aberration-time, which is different for each satellite : $\tau = 498^s.46 . \Delta'$. Further, $L' = L - \epsilon'$ where $\epsilon' = \tau \frac{dL}{dt}$ is the correction for the motion of Jupiter [see *M.N.*, lxxi. p. 93, or *Cape Annals*, xii. 1, pp. 100-101]. The values of L, B , and Δ refer to the position of the earth at the time t_0 and Jupiter at the time t . Thus to the value of $\log \Delta$ interpolated from the *Nautical Almanac* for the time t_0 must be added the correction $\delta \log \Delta$ given in *M.N.*, lxxi. p. 94. L and B are computed by the formulas (16) and (17), *i.e.*, pp. 93 and 94. In (16) we must take for $\odot$ the sun's longitude taken from the *N.A.* for time t_0 + aberration - nutation, and $l\psi$ should be taken for the time t . The position-angle P is computed by

$$\tan \phi = \tan I \sin (L - N)$$

$$\tan P = - \sin \phi \sec (B - \phi) \cot (L - N),$$

where I and N are the inclination and descending node of the earth's true equator on the fundamental plane. An abridged ephemeris of L', B, P and $\log \Delta$ is given below, for Greenwich mean time 12^h for April and May, and 8^h for August and September. The values of L', B , and Δ include all the corrections mentioned above, as well as the correction for parallax for a mean hour-angle. The table is so arranged that it must be entered with the argument t , *i.e.* the time corrected for aberration.

		L.	B.	P.	Log Δ .
Apr.	20 ^h 12	287° 51' 79	- 1° 7' 35.3	351° 7' 43	0° 69' 03.93
May	2	287° 8' 47.3	- 1° 7' 04.7	350° 59' 3	0° 67' 38.80
	14	287° 7' 24.8	- 1° 6' 83.5	351° 2' 13	0° 65' 82.34
	26	287° 1' 60.0	- 1° 6' 71.7	351° 16' 55	0° 64' 42.21
Aug.	19 8	278° 4' 10.8	- 1° 6' 82.1	355° 12' 27	0° 64' 74.21
	31	278° 0' 39.2	- 1° 6' 70.2	355° 22' 39	0° 66' 16.19
Sept.	12	278° 1' 24.6	- 1° 6' 51.2	355° 20' 14	0° 67' 71.59
	24	278° 6' 61.5	- 1° 6' 24.9	355° 5' 18	0° 69' 32.17

The co-ordinates measured on the plate must be corrected for aberration and refraction before they are compared with the tabular co-ordinates. Both corrections are of the form $\delta x = A x$,* and are easily computed. In computing the correction for refraction, it is sufficiently accurate to use a mean value for the temperature and the atmospheric pressure. It has been verified that the errors thus made never exceed 0'00003 even in extreme cases.

Instead of the corrections Δl_i to the longitudes of the satellites, I have introduced as unknowns

$$\lambda_i = \frac{1}{k_i} \Delta l_i,$$

where

$$k_1 = 150, \quad k_2 = 100, \quad k_3 = \frac{200}{3}, \quad k_4 = \frac{100}{3}.$$

The coefficients are thus

$$\frac{\partial x}{\partial \lambda} = a = \frac{a' \rho}{\Delta'} k \sin i^\circ \cos (v - L').$$

Further, we must, to eliminate the effect of scale-value, introduce the unknown μ , of which the coefficient is

$$\frac{\partial x}{\partial \mu} = x.$$

* The effect of refraction on the co-ordinates x and y depends on how the plate is oriented. Let P_t be the true and P_a the apparent angle between the projection on the plate of Jupiter's axis and the declination-circle through Jupiter. Then we can orient the plate for the measures either in the angle P_t or P_a . In the first case we find for the corrections of refraction

$$\delta x = A x - B y \qquad \delta y = A' y + B x,$$

and in the second case

$$\delta x = A x \qquad \delta y = A y - D x.$$

The values of A , A' , B , D are, if k is the constant of refraction :

$$\begin{aligned} A &= k \{ 1 + \tan^2 \zeta \sin^2 (P - q) \} \\ A' &= k \{ 1 + \tan^2 \zeta \cos^2 (P - q) \} \\ B &= k \{ \tan \zeta \sin q \tan \delta - \tan^2 \zeta \sin (P - q) \cos (P - q) \} \\ D &= 2k \tan^2 \zeta \sin (P - q) \cos (P - q). \end{aligned}$$

The first case was used in my measures of the Cape plates of 1891, 1903, and 1904, described in *Cape Annals*, xii. 3 (p. 34, with $P = 90^\circ + p$), and *Gron. Publ.*, xvii. (p. 67, with $P = 0^\circ$). Evidently the second method is preferable for the co-ordinates x , and it has been used in the measures here described.

The unknowns λ_i contain the effect of corrections to the mean longitudes and to the inequalities of long periods. Of the inequalities with short periods the most important are the equations of the centre and the great inequalities. If the corresponding corrections are denoted by δh , δk , and δx , then the correction to the co-ordinate x becomes

$$-f - \frac{1}{3}K\{\cos(2l-L)\delta h - \sin(2l-L)\delta k \mp \sin(2l-L+u)\delta x\},$$

where

$$K = \frac{3}{4} \frac{a'}{\Delta} \sin 1^\circ, \quad u = l_1 - 2l_2 = l_2 - 2l_3 + 180^\circ$$

$$f = K\{\cos L\delta h - \sin L\delta k \mp \sin(L+u)\delta x\}$$

[see also *Cape Annals*, xii. 1, p. 156, and *Gron. Publ.*, xvii. p. 49]. The term with δx is absent for the fourth satellite, and of the double sign the lower must be taken for Satellite II. The angles L and u change very slowly, and for the time covered by one series f can consequently be treated as a constant. The second term is small and periodic, and in the result from a series of observations distributed with any approach to regularity over a few revolutions of the satellites, its effect will be entirely eliminated. It will, however, increase the residuals. We have thus, if we neglect this small periodic term,

$$x_{\text{obs.}} - x_{\text{tab.}} = a\lambda + \mu x - f.$$

Thus if we put

$$x_i - x_o = x_i, \quad -f_i + f_o = c_i, \quad x_{\text{obs.}} - x_{\text{tab.}} = n,$$

then the equations of condition derived from the measures of one plate are

$$\begin{aligned} \frac{3}{4}a_1\lambda_1 - \frac{1}{4}a_2\lambda_2 - \frac{1}{4}a_3\lambda_3 - \frac{1}{4}a_4\lambda_4 + x_1\mu + c_1 &= n_1 \\ -\frac{1}{4}a_1\lambda_1 + \frac{3}{4}a_2\lambda_2 - \frac{1}{4}a_3\lambda_3 - \frac{1}{4}a_4\lambda_4 + x_2\mu + c_2 &= n_2 \\ -\frac{1}{4}a_1\lambda_1 - \frac{1}{4}a_2\lambda_2 + \frac{3}{4}a_3\lambda_3 - \frac{1}{4}a_4\lambda_4 + x_3\mu + c_3 &= n_3 \\ -\frac{1}{4}a_1\lambda_1 - \frac{1}{4}a_2\lambda_2 - \frac{1}{4}a_3\lambda_3 + \frac{3}{4}a_4\lambda_4 + x_4\mu + c_4 &= n_4 \end{aligned}$$

with the condition

$$c_1 + c_2 + c_3 + c_4 = 0.$$

The weight of each of these equations is $\frac{4}{3}$. The normal equations thus become

$$\begin{aligned} [a_1^2]\lambda_1 - \frac{1}{3}[a_1a_2]\lambda_2 - \frac{1}{3}[a_1a_3]\lambda_3 - \frac{1}{3}[a_1a_4]\lambda_4 &= \Lambda_1 \\ [a_2^2] & - \frac{1}{3}[a_2a_3] & - \frac{1}{3}[a_2a_4] &= \Lambda_2 \\ [a_3^2] & - \frac{1}{3}[a_3a_4] &= \Lambda_3 \\ [a_4^2] &= \Lambda_4 \end{aligned}$$

$$\frac{4}{3}[x^2]\mu + \sum_{i=1}^4 [x_i]c_i + \sum_{i=1}^4 [a_i x_i]\lambda_i = \frac{4}{3}[xn]$$

$$\frac{4}{3}Nc_i + \frac{4}{3}[a_i]\lambda_i + \frac{4}{3}[x_i]\mu = \frac{4}{3}[n_i]$$

where N is the number of plates, and

$$\Lambda_i = \frac{4}{3}[a_i n_i] - \frac{4}{3}[a_i x_i]\mu - \frac{4}{3}[a_i]c_i.$$

From these equations the unknowns λ_i , c_i , and μ can easily be solved. In deriving them the assumption is made that by the use of the standard stars all plates have effectively been brought to one and the same scale-value, so that there is only one μ common to all plates. It was, however, very soon realised that this is not the case. The residuals remaining after substitution of the unknowns were for the majority of the plates of both series very much larger than could be reasonably expected, and in most cases they were approximately proportional to x_i . It thus became evident that every plate has its own scale-value even after the reduction by means of the standard stars, and a separate value of μ must be derived for each plate. As the point is important, the discussion of the plates of the second series will be related to some length. In the first solution, A, only one value of μ was introduced. The resulting values of λ_i and c_i are given below. The average residual of this solution was ± 0.00093 .* Then from these residuals corrections $\delta\mu$ were derived for each plate by the formula

$$\delta\mu = \frac{\sum x_i \Delta_i}{\sum x_i^2}.$$

Thereby the average residual was at once brought down to ± 0.00042 . From these residuals further corrections to λ_i and c_i were derived, which, added to those from solution A, give the values B_1 which are stated below. The comparison of the values A and B_1 shows that the values of λ_i and c_i are practically not affected by the large corrections for scale-value applied to the individual plates. In order to establish this important fact with still greater certainty it was decided to make a third solution in which the order of approximation is reversed. Thus in the solution B_2 I first derived for every plate a value of μ directly from the original right-hand members n , without introducing either λ_i or c_i . The average residual then became ± 0.00106 . Next the unknowns c_i were introduced, and from the residuals corrections to μ were determined and then again to c_i until the best solution was attained which is possible without introducing the λ 's. The average residual had then become ± 0.00081 . Now the unknowns λ_i were introduced and thereby, without further corrections to either μ or c_i , the average residual was brought down to ± 0.00058 . The solution was then completed by successive approximations to all unknowns. The final values of λ_i and c_i are given below. The final values of μ were practically the same as those of $\mu + \delta\mu$ found in solution B_1 . The final result is thus the same, independent of the method of approximation applied. The most important difference between the solutions B_1 and B_2 is that in B_1 the process of successive approximations converged rapidly and in B_2 very slowly. The results from the different solutions will now be given. The solution C will be presently explained.

* The average value of the original right-hand members n is ± 0.00203 .

Solution.	A.	B ₁ .	B ₂ .	C.
λ_1	+ 0'000450	+ 0'000442	+ 0'000401	+ 0'000423
λ_2	- 272	- 242	- 232	- 204
λ_3	+ 376	+ 426	+ 424	+ 425
λ_4	+ 192	+ 192	+ 209	+ 141
c_1	- 0'00121	- 0'00097	- 0'00094	- 0'00103
c_2	+ 88	+ 83	+ 74	+ 89
c_3	+ 25	+ 26	+ 37	+ 22
c_4	+ 8	- 12	- 17	- 8
Average residual }	$\pm$ '00093	$\pm$ '00041	$\pm$ '00040	$\pm$ '00043

Now if we accept the conclusion that the scale-value determined from the standard stars cannot be used for the satellites, then the question arises whether the same thing must not be true of the orientation. The effect of the orientation on the co-ordinates x is very small, since the largest value of y is only about 0'25; but a systematic error in orientation would be very grave, since the principal term in y , viz. $\sin B \cos (v - L')$ is proportional to the coefficient of λ . It was therefore decided to measure also the co-ordinates y . Every plate was thus again put in the instrument, and after ascertaining the difference of the index-correction K with its value for the first measure, was brought to the position-angle $P_0 + 90^\circ$. The images of the four satellites produced by one exposure are then arranged in a line parallel to the guiding cylinder and the differences of the y 's can be measured by the micrometer-screw, without referring to the scale at all. Then from the residuals

$$n' = (y_i - y_0)_{\text{obs.}} - (y_i - y_0)_{\text{tab.}},$$

corrected for refraction when necessary, we derive a correction of orientation for every plate by the formula

$$B = \frac{\sum n'_i x_i}{\sum x_i^2}.$$

The average value of n' was $\pm 0'0029$, and the residuals $\Delta'_i = n'_i - Bx_i$ amounted only to $\pm 0'0009$ on the average. This proves the reality of the corrections B . They were of the same order of magnitude as the μ 's. The correction to be applied to the observed co-ordinates x_i is now

$$\delta x_i = By_i.$$

These corrections were applied and a new solution was made, which is the solution C already given above. It will be seen that, with the exception of λ_4 , the introduction of the corrections B has had practically no influence on the unknowns. Here again we can say that approximately the orientation for the three inner satellites

is derived from the fourth. By various discussions [see, *e.g.*, *M.N.*, lxxi. pp. 98, 596; *Proceedings Acad. Amst.*, xviii. p. 1012] it has appeared that my theory of the fourth satellite, both in longitude and latitude, is in excellent agreement with the observations. The corrections derived from that satellite can thus be accepted with confidence.

The discussion of the plates of the second series has here been related somewhat in detail, but the first series might as well have been taken as example, as it leads to exactly the same results. The separate values of μ and B are given in the following table, which also contains the corrections for scale-value a which were derived from the standard stars.

Plate.	μ .	a .	B.	Plate.	μ .	a .	B.
4030	- '00025	+ '00090	+ '00069	4110	- '00026	+ '00042	+ '00073
4034	- 37	+ 58	+ 49	4112	+ 32	+ 43	+ 60
4036	- 40	+ 72	+ 245	4113	- 25	+ 41	+ 11
4037	+ 3	+ 81	+ 74	4115	+ 35	+ 44	+ 75
4041	- 24	+ 42	+ 307	4118	+ 6	+ 40	+ 76
4042	- 19	+ 45	+ 57	4119	+ 33	+ 41	+ 71
4043	- 16	+ 49	+ 85	4123	+ 33	+ 37	+ 65
4044	- 12	+ 44	+ 291	4125	+ 29	+ 41	+ 82
4045	- 2	+ 37	+ 115	4126	- 30	+ 31	+ 52
4047	- 50	+ 17	- 221	4127	- 18	+ 37	+ 85
4048	- 9	+ 49	+ 95	4128	- 56	+ 35	+ 129
4049	- 22	+ 53	+ 89	4129	- 44	+ 35	+ 66
4050	- 20	+ 48	+ 89	4130	- 74	+ 31	+ 153
4053	- 74	+ 44	+ 9	4131	- 45	+ 40	+ 154
4054	- 27	+ 45	+ 131	4132	- 7	+ 34	+ 90
				4134	- 61	+ 38	+ 158
Mean	- '000217	+ '000515	+ '00099		- '000137	+ '000381	+ '00088
Average deviation from mean	$\pm$ 164	$\pm$ 128	$\pm$ 79		$\pm$ 321	$\pm$ 28	$\pm$ 33

There can be hardly any doubt on the reality of the individual corrections μ and B. It is, however, not easy to see how they originate. The standard stars are very near the edges of the plate, and it might be thought possible that so near the edge distortions of the photographic film had occurred of an amount which, according to general experience, never occurs nearer the centre. If this were the true explanation, the values of a should show the effect of these distortions, and it would be eliminated again by the introduction of the corrections μ derived from the satellites, so that $a + \mu$ would be free from it. Now the fact is that a is much more approximately constant than either μ or $a + \mu$. This explanation thus becomes extremely improbable. I am unable, however, to offer a better one. We must simply accept the facts as they

are. Four solutions were made, as has been explained (above, viz.:

- A : orientation and scale-value from standard stars,
 $\left. \begin{matrix} B_1 \\ B_2 \end{matrix} \right\}$: orientation from stars, scale-value from satellites,
C : orientation and scale-value from satellites,

and the results from these four solutions are practically* the same. This is the most important conclusion from the preceding discussions.

The solution C has been finally adopted. It is the mean of two solutions C_1 and C_2 . In C_1 all plates were included with equal weights, in C_2 the two plates 4118 and 4132 were rejected. In the mean these two plates are thus included with half weight. The two solutions were

	C_1 .	C_2 .
λ_1	+ 0'000422	+ 0'000424
λ_2	- 186	- 222
λ_3	+ 412	+ 438
λ_4	+ 137	+ 145
c_1	- 0'00104	- 0'00102
c_2	+ 89	+ 88
c_3	+ 22	+ 23
c_4	- 7	- 9

The first series was treated in the same way, but it is not necessary to relate the discussions at the same length. Two final solutions were made of which the first, A, included all plates, and in the other, B, the plates 4053 and 4054 were rejected. It is probable that the very large residuals* of these plates are due to nothing else but exceptionally large accidental errors of the images on the plate. It was decided to adopt the mean of the two solutions. The resulting values of μ and B have already been given, those of λ_i and c_i are

	A.	B.	Mean.
λ_1	+ 0'000428	+ 0'000494	+ 0'000461
λ_2	+ 289	+ 287	+ 278
λ_3	+ 333	+ 342	+ 338
λ_4	+ 171	+ 163	+ 167
c_1	- 0'00116	- 0'00134	- 0'00125
c_2	+ 93	+ 113	+ 103
c_3	+ 2	+ 1	+ 2
c_4	+ 21	+ 20	+ 20

The residuals from both solutions are practically the same.

* The sum of the squares of the residuals (not multiplied by the weight) of the first thirteen plates is '00001373, and of the last two alone '00001326. Both plates were measured twice, and the two measures are in close agreement with each other.

The problem how to derive the mean error of the unknowns from the residuals is beset with considerable difficulty. The unknowns determined from the equations are :

Four λ 's	4
Four c 's with one condition	3
One μ for each plate	15

or 22 in all. But each equation contains only *one* μ , and therefore only eight unknowns. Thus if in the usual formula

$$\rho^2 = \frac{[p\Delta\Delta]}{n-m}$$

we take $m=22$, we certainly get a value of μ which is much too large, and probably $m=8$ will give a better approximation to the truth. On the other hand, the weights of the unknowns λ_i as derived from the normal equations in the form in which they have been used for the solution, take no account of the c 's nor of the fifteen μ 's. Now the effect of the c 's, and of the *one* μ supposed to be the same for all plates, on the weight of the λ 's is certainly very small, but the effect of the fifteen individual μ 's in depressing the weight of the λ 's and c 's is probably very noticeable. The point is not of enough importance to justify a detailed investigation, and I have simply combined the too large weight with the too large mean error of weight unity, hoping thus by making two errors in opposite directions to get a nearly correct result. The mean error of weight unity has been derived both from the squares of the residuals and from their average numerical values. I find

From average residuals	$\rho = \pm 0.00083$
From squares	± 84

Similarly, for the second series we have

From average residuals	$\rho = \pm 0.00078$
From squares	± 77

In both cases I adopt the largest value. The mean errors of the unknowns then become

	First Series.	Second Series.
λ_1	± 0.00057	± 0.00058
λ_2	± 56	65
λ_3	± 51	49
λ_4	± 60	55
c_i	± 0.00019	± 0.00017

Returning now to the original unknowns, we have finally

$$1913.3403 = \text{May } 4.354 \left\{ \begin{array}{l} \Delta l_1 = +0.0691 \pm 0.0075 \\ \Delta l_2 = +0.278 \pm 56 \\ \Delta l_3 = +0.225 \pm 35 \\ \Delta l_4 = +0.056 \pm 20 \end{array} \right.$$

$$1913\cdot6883 = \text{Sept. } 8\cdot843 \left\{ \begin{array}{l} \Delta l_1 = +0\cdot0634 \pm 0\cdot0077 \\ \Delta l_2 = -0\cdot0204 \pm 65 \\ \Delta l_3 = +0\cdot0283 \pm 32 \\ \Delta l_4 = +0\cdot0047 \pm 18 \end{array} \right.$$

If there were no other accidental errors than the pure error of pointing, the mean error of one right-hand member n_i would be for the first series $\frac{1}{2}\zeta$, and for the second, of which all plates were measured twice, $\zeta/2\sqrt{2}$. The values of ζ which were derived above are average errors, and refer to the differences n_i , which have the weight $\frac{4}{3}$. Thus to find the mean error of unit weight they must further be multiplied by $1\cdot253 \cdot \sqrt{\frac{4}{3}} = 1\cdot44$. If we adopt $\zeta = \pm 7\cdot0$ for both series, we thus find for the two series

$$\rho = \pm 0\cdot00050 \quad \text{and} \quad \rho = \pm 0\cdot00036.$$

The values derived from the residuals were

$$\pm 0\cdot00084 \quad \text{and} \quad \pm 0\cdot00078.$$

The excess is due to two causes: firstly, the errors of the photographic images produced by atmospheric, instrumental, and photographic influences; and secondly, the effect on the residuals of that part of the remaining uncertainties of the theory which is not absorbed in the unknowns c_i . The combined effect of these two causes is thus for the two series found to be equivalent to a mean error of

$$\pm 0\cdot00068 \quad \text{and} \quad \pm 0\cdot00069$$

respectively. This error being so much larger than that due to the pointings alone, it is evident that the gain in accuracy obtained by the double measures of the second series is not proportional to the labour bestowed on it. It is therefore proposed to measure future series only once in each position of the plates, as was done with the first of the two series discussed here.

The corrections Δl_i resulting from the solutions include the libration and the effect of corrections to the mean longitudes and to the coefficients of inequalities of long periods. The libration was not included in the tabular places. The long periodic inequalities depend on the excentricities, which are still subject to considerable uncertainty, especially that of Satellite II. The discussion of the results can of course not be undertaken now, but must be deferred till they can be combined with those from the whole series, extending over eight years, according to the programme given in the *History of the Cape Observatory*. At present we can only conclude that indeed it is possible, from a series of fifteen plates distributed over a month's time, to derive reliable corrections to the longitudes of the satellites. Although the accuracy of the results as shown by the mean errors does perhaps not quite reach the standard which was expected when

that programme was drawn up, the result of the discussion must on the whole be considered very satisfactory.

The following tables contain for every plate in the first column the number of the plate, the initials of the observer, and the definition as estimated by him. The observers were :

JL = Dr. J. Lunt.	JJ = J. W. Jackson.
RW = R. Woodgate.	GD = G. Duncan.

The second column contains the date and the Greenwich mean time, not corrected for aberration, *i.e.* the time t_0 . The fourth to eighth columns give the longitudes, radii vectores, and co-ordinates of the satellites and the coefficients $\alpha_i = \frac{\partial x_i}{\partial \lambda_i}$. The ninth column contains the observed difference of the x -co-ordinate of each satellite from the mean of the four. These include the corrections for refraction, aberration, orientation from the satellites (*i.e.* the corrections by explained above), and scale-value from the standard-stars $[\alpha x]$, but not for scale-value from the satellites $[\mu x]$. The tenth column gives the right-hand members of the equations of condition $n = x_{\text{obs.}} - (x_i - x_o)$, and the eleventh contains the final residuals. In the column of remarks, those printed in italics were made by the observers at the Cape during the exposures, and the others by the observers at Leiden during the measures.

Plate No. Obsr. Defn.	Date 1913. G.M.T.	Satellite.	η .	ρ .	α .	x .	y .	z .	Resid.	Remarks.
4030 JL I	Apr. 20 h m s 15 28 6	I.	257°4360	1°003675	-0°09467	-0°0508	+4°49	+1°41415	+°00007	-°00055
		II.	159°5549	0°999395	-2°48069	+°0681	-3°38	-0°07257	-63	8
		III.	202°0944	1°002067	-5°01335	-°0237	+0°47	-2°60519	-59	+38
		IV.	279°9874	0°992696	-1°14629	-°2641	+5°04	+1°26362	+116	+26
4034 JL I	Apr. 22 h m s 15 54 53	I.	307°9505	1°001588	+0°69271	-0°0565	+4°89	-1°91646	+°00086	+47
		II.	6°0579	1°004705	+3°11689	-°0381	+1°11	+0°50598	-88	-69
		III.	303°5162	0°999042	+1°58143	-°1506	+5°64	-1°22781	+79	-31
		IV.	324°1641	0°993892	+5°24909	-°2521	+4°12	+2°63827	-79	+51
4036 JL 1-2	Apr. 26 h m s 15 55 6	I.	42°2789	0°996407	+1°82758	+0°0237	-2°18	-0°42342	-°00007	+87
		II.	50°9635	1°008666	+2°70809	+°2702	-3°10	+0°45763	+47	-35
		III.	145°1738	1°001626	-3°11887	+°1297	-4°74	-5°36867	+113	-40
		IV.	50°7332	1°003700	+7°58693	+°1053	-2°86	+5°33447	-153	-4
4037 JL 3-4	Apr. 28 h m s 15 14 10	I.	83°8237	0°996565	+0°82169	+0°0546	-4°84	+1°39269	-°00286	+28
		II.	250°8425	0°991374	-1°92199	-°0535	+4°46	-1°34528	+285	+62
		III.	244°0308	1°001027	-3°56090	-°1189	+4°33	-2°98620	+84	-58
		IV.	92°7926	1°007034	+2°36574	+°2541	-5°14	+2°93877	-83	-30
4041 RW 1-2	Apr. 30 h m s 15 41 18	I.	134°9033	0°999218	-0°92916	+0°0544	-4°76	-0°90479	-°00335	-121
		II.	94°7176	1°006988	+0°74250	+°0768	-5°56	+0°77108	+86	+87
		III.	345°7627	0°997789	+4°37816	-°0926	+3°19	+4°40665	+77	-8
		IV.	135°7178	1°006735	-4°30235	+°2718	-4°73	-4°27292	+171	+42

Images faint and hazy.

Images very hazy.

Image b of standard stars
double.

Plate No. Obsr. Defn.	Date 1913. G. M. T.	Satellite.	ϵ .	ρ .	r .	γ .	α .	$\chi_{\text{obs.}}$	n .	Resid.	Remarks.
4042 JL 1-2	May h m s 15 18 34	I. II. III. IV.	334°3424 194°4308 35°4894 156°6961	0°999435 0°993544 0°998286 1°003175	+1°48425 -3°23187 +4°94679 -6°93223	-0°0426 +0°278 +0°581 +2°249	+3°69 -0°34 -1°83 -3°52	+2°41729 -2°29792 +5°87837 -5°99773	-0°0023 +68 -169 +123	-0°0011 53 11 54	
4043 RW 3	May h m s 16 12 4	I. II. III. IV.	186°1608 301°1650 87°7895 178°8920	1°002455 0°996094 0°999846 1°002786	-2°01725 +0°74939 +1°79081 -8°70556	+0°0135 -0°875 +1°490 +1°428	-1°08 +5°52 -5°66 -1°74	+0°02737 +2°79711 +3°83411 -6°65858	-0°0103 +207 -235 +133	+33 27 13 7	
4044 RW 3	May h m s 14 1 34	I. II. III. IV.	214°5501 132°8677 183°6320 220°0685	1°003565 1°001941 1°002059 0°997646	-1°98650 -1°39417 -5°09704 -8°51237	-0°0163 +0°865 +0°502 -0°447	+1°56 -5°21 -1°50 +2°02	+2°26093 +2°85251 -0°85003 -4°26340	-0°0009 -84 51 +145	+42 19 39 14	<i>Last half cloudy.</i> All images hazy; b and c faint, a not measurable. Only one image of stand- ard stars.
4045 RW 3	May h m s 11 28 27	I. II. III. IV.	130°7303 270°6052 70°0950 326°9436	0°999361 0°994142 0°999233 0°993946	-0°81622 -0°98139 +3°26008 +5°85921	+0°0572 -0°743 +1°311 -2°554	-5°06 +5°53 -4°90 +4°20	-2°64982 -2°80873 +1°42920 +4°02935	-0°0318 +308 -46 +56	-29 12 81 63	Images a hazy.
4047 RW 3	May h m s 11 47 21	I. II. III. IV.	320°5821 3°0818 113°3412 138°9137	0°999581 1°006260 1°000625 1°006731	+1°16644 +3°32654 -0°54391 -5°02290	-0°0533 +0°450 +1°569 +2°759	+4°71 +1°51 -6°32 -4°81	+1°43582 +3°59443 -0°27765 -4°75261	+0°0092 -57 -220 +183	+68 26 37 4	Bad images, especially b .

	May 19	I.	7 ^h 17 ^m 50 ^s	0 ^h 99 ^m 69 ^s 11	+ 2 ^h 11 ^m 69 ^s 5	- 0 ^h 01 ^m 27	+ 1 ^h 01	+ 6 ^h 07 ^m 38 ^s 6	- 0 ^h 02 ^m 02	-	49	Cloud about.
4048 RW 2-3	h m s 11 43 11	II.	204 ^h 524 ^m 6	0 ^h 99 ^m 18 ^s 38	- 3 ^h 38 ^m 12 ^s 7	+ 0 ^h 13 ^m 2	+ 0 ^h 72	+ 0 ^h 57 ^m 8 ^s 63	+ 97	+	13	
		III.	213 ^h 505 ^m 1	1 ^h 00 ^m 18 ^s 30	- 5 ^h 27 ^m 53 ^s 4	- 0 ^h 56 ^m 1	+ 1 ^h 76	- 1 ^h 31 ^m 46 ^s 0	+ 181	+	131	
		IV.	181 ^h 584 ^m 3	1 ^h 00 ^m 26 ^s 02	- 9 ^h 29 ^m 50 ^s 7	+ 1 ^h 35 ^m 0	- 1 ^h 55	- 5 ^h 33 ^m 7 ^s 89	- 75	-	97	
4049 JL I	May 20	I.	251 ^h 811 ^m 4	1 ^h 00 ^m 32 ^s 70	- 1 ^h 26 ^m 67 ^s 1	- 0 ^h 05 ^m 04	+ 4 ^h 62	+ 1 ^h 19 ^m 22 ^s 1	- 0 ^h 01 ^m 91	-	106	
	h m s 16 33 58	II.	327 ^h 665 ^m 9	1 ^h 02 ^m 21 ^s 13	+ 2 ^h 22 ^m 64 ^s 0	- 0 ^h 23	+ 4 ^h 60	+ 4 ^h 68 ^m 7 ^s 54	+ 30	+	45	
		III.	273 ^h 858 ^m 8	1 ^h 00 ^m 11 ^s 1	- 1 ^h 29 ^m 34 ^s 1	- 1 ^h 55	+ 6 ^h 20	+ 1 ^h 16 ^m 8 ^s 24	+ 81	+	36	
		IV.	207 ^h 469 ^m 3	0 ^h 99 ^m 19 ^s 3	- 9 ^h 50 ^m 50 ^s 6	+ 0 ^h 07 ^m 0	+ 0 ^h 97	- 7 ^h 04 ^m 7 ^s 97	+ 79	+	26	
4050 RW 1-2	May 21	I.	55 ^h 985 ^m 5	0 ^h 99 ^m 63 ^s 71	+ 1 ^h 69 ^m 24 ^s 4	+ 0 ^h 03 ^m 83	- 3 ^h 53	+ 2 ^h 03 ^m 51 ^s 0	- 0 ^h 02 ^m 69	+	43	
	h m s 11 53 23	II.	48 ^h 235 ^m 8	1 ^h 00 ^m 86 ^s 45	+ 2 ^h 99 ^m 40 ^s 8	+ 0 ^h 24 ^m 0	- 3 ^h 11	+ 3 ^h 33 ^m 8 ^s 92	- 51	+	21	
		III.	314 ^h 425 ^m 0	0 ^h 99 ^m 87 ^s 38	+ 2 ^h 49 ^m 23 ^s 6	- 1 ^h 37 ^m 1	+ 5 ^h 70	+ 2 ^h 83 ^m 8 ^s 59	+ 88	-	31	
		IV.	224 ^h 907 ^m 1	0 ^h 99 ^m 69 ^s 65	- 8 ^h 56 ^m 03 ^s 0	- 0 ^h 82 ^m 1	+ 2 ^h 59	- 8 ^h 21 ^m 26 ^s 2	+ 233	-	24	
4053 RW 3	May 23	I.	123 ^h 129 ^m 6	0 ^h 99 ^m 54 ^s 6	- 0 ^h 59 ^m 41 ^s 5	+ 0 ^h 06 ^m 14	- 5 ^h 50	- 0 ^h 39 ^m 57 ^s 0	- 0 ^h 04 ^m 48	-	158	Bad plate.
	h m s 14 13 36	II.	261 ^h 607 ^m 6	0 ^h 99 ^m 33 ^s 10	- 1 ^h 49 ^m 60 ^s 5	- 0 ^h 68 ^m 7	+ 5 ^h 42	- 1 ^h 28 ^m 80 ^s 2	+ 510	+	133	
		III.	60 ^h 352 ^m 0	0 ^h 99 ^m 87 ^s 81	+ 4 ^h 05 ^m 04 ^s 6	+ 1 ^h 19 ^m 2	- 4 ^h 40	+ 4 ^h 24 ^m 90 ^s 7	- 432	-	6	
		IV.	270 ^h 638 ^m 8	0 ^h 99 ^m 29 ^s 63	- 2 ^h 77 ^m 19 ^s 6	- 2 ^h 62 ^m 4	+ 5 ^h 38	- 2 ^h 56 ^m 53 ^s 5	+ 368	+	33	
4054 RW 2-3	May 25	I.	143 ^h 616 ^m 5	1 ^h 00 ^m 99 ^s 6	- 1 ^h 30 ^m 30 ^s 4	+ 0 ^h 05 ^m 21	- 4 ^h 63	- 1 ^h 21 ^m 92 ^s 1	- 0 ^h 02 ^m 76	-	102	Images not good.
	h m s 11 6 5	II.	89 ^h 794 ^m 6	1 ^h 00 ^m 56 ^s 64	+ 1 ^h 05 ^m 22 ^s 5	+ 0 ^h 76 ^m 9	- 5 ^h 85	+ 1 ^h 14 ^m 05 ^s 9	+ 175	+	177	
		III.	154 ^h 375 ^m 9	1 ^h 00 ^m 17 ^s 89	- 4 ^h 09 ^m 22 ^s 2	+ 1 ^h 00 ^m 7	- 4 ^h 42	- 4 ^h 00 ^m 51 ^s 4	+ 49	-	21	
		IV.	311 ^h 571 ^m 9	0 ^h 99 ^m 29 ^s 55	+ 3 ^h 99 ^m 66 ^s 6	- 2 ^h 90 ^m 8	+ 5 ^h 14	+ 4 ^h 08 ^m 37 ^s 6	+ 51	-	54	

Plate No. Obs. Defn.	Date 1913. G.M.T.	Satellite.	v.	p.	x.	y.	a.	x _{obs.}	n.	Resid.	Remarks.
4110 JL I	Aug. 19	I.	328.8203	0.996256	+1.67471	-0.0416	+3.62	+0.65887	-0.0030	-0.0026	<i>Wind shaking instrument. Clouded up for 2nd Star exposure. Bad images.</i>
	h m s	II.	151.6462	0.991753	-2.76018	+0.0696	-3.60	-3.77273	+299	-36	
	7 2 57	III.	153.1038	1.002107	-4.53341	+0.841	-3.74	-5.54710	+185	+46	
		IV.	3.5597	0.997648	+9.68103	-0.821	+0.48	+8.66095	-454	+15	
4112 JJ 2-3	Aug. 29	I.	196.4994	1.002413	-2.10602	-0.0076	+0.82	-0.13316	-0.0234	-57	<i>Cloudy.</i>
	h m s	II.	81.1481	0.996711	+0.98136	+0.731	-5.63	+2.95793	+137	+38	
	6 14 53	III.	294.3037	0.998793	+1.50273	-1.485	+6.01	+3.47970	+117	+1	
		IV.	216.8930	0.998007	-8.27885	-0.822	+2.65	-6.30447	-82	+20	
4113 JL 2	Aug. 30	I.	45.9315	0.999313	+1.67312	+0.0367	-3.41	+2.31804	-0.0448	-51	
	h m s	II.	186.5727	0.993563	-3.34725	+0.242	-0.15	-2.69514	+271	+13	
	6 51 48	III.	346.0948	0.997900	+4.97226	-0.474	+2.33	+5.61959	-207	+32	
		IV.	239.1652	0.995396	-5.89573	-1.767	+4.25	-5.24250	+383	+7	
4115 JJ & GD 2-3	Sept. 1	I.	84.2185	1.001838	+0.50391	+0.0590	-5.37	-0.89786	-0.0209	+37	
	h m s	II.	23.6400	1.004469	+3.24190	+0.008	-1.58	+1.84453	+233	+6	
	5 50 53	III.	84.8523	1.000210	+1.22018	+1.543	-6.06	-0.18102	-152	-17	
		IV.	281.9292	0.992583	+0.63274	-2.228	+5.40	-0.76565	+129	-25	
4118 JJ 3-4	Sept. 3	I.	137.2214	1.003708	-1.32886	+0.0478	-4.27	-0.70444	-0.0240	-26	<i>Bad defn. and cloud. Very bad plate.</i>
	h m s	II.	231.2758	0.999576	-2.42317	-0.400	+3.98	-1.88381	+254	+145	
	6 36 1	III.	186.7373	1.002056	-5.32316	-0.088	-0.14	-4.78554	+80	-92	
		IV.	326.3737	0.993835	+6.92793	-2.207	+3.58	+7.46381	-94	-28	

4110 JL 1 2	Sept. 4 h m s 5 31 52	I. 331°6405 327°1185 234°6621 347°2009	0°996327 1°008236 1°000876 0°995785	+1°67369 +2°52919 -3°63558 +8°66319	-0°0369 -0°566 -1°211 -1°474	+3°23 +3°82 +4°48 +1°92	-0°63425 +0°21998 -5°94168 +6°35595	-0°0032 -159 +155 +38	+8 110 51 50
4123 JJ 3	Sept. 6 h m s 5 32 59	I. 19°0430 170°4465 335°4965 30°4969	0°998002 0°992738 0°997919 1°001232	+2°03270 -3°12496 +4°42675 +8°57722	+0°0102 +0°448 -0°728 +0°501	-1°04 -1°73 +3°28 -2°06	-0°94600 -6°10284 +1°45022 +5°59863	-0°0077 +5 +140 -66	+100 55 2 43
4125 JJ 3	Sept. 8 h m s 5 25 7	I. 64°9280 11°4663 76°1618 73°1993	1°000995 1°005262 0°999918 1°006033	+1°12839 +3°29277 +1°95326 +3°90033	+0°0491 -0°167 +1°449 +2°166	-4°53 -0°34 -5°66 -4°90	-1°44167 +0°72656 -0°61699 +1°33210	-0°0137 +248 -156 +46	+40 29 65 2
4126 GD 3-4	Sept. 10 h m s 8 18 50	I. 136°0259 228°0814 182°5295 118°3038	1°003730 0°999899 1°002217 1°007487	-1°26629 -2°49737 -5°19216 -3°19611	+0°0475 -0°349 +0°027 +2°648	-4°25 +3°66 -0°59 -5°05	+1°76833 +0°54250 -2°15195 -0°15886	-0°0336 +188 +223 -73	31 114 10 92
4127 JL 2	Sept. 11 h m s 6 48 17	I. 326°7452 322°0597 229°5166 138°2325	0°996330 1°008213 1°001070 1°006667	+1°52810 +2°27450 -3°89036 -5°93220	-0°0397 -0°718 +1°087 +2°357	+3°52 +4°12 +4°00 -4°09	+3°03237 +3°77677 -2°38296 -4°42618	-0°0072 -272 +241 +103	+70 63 13 4
4128 JJ 2-3	Sept. 12 h m s 9 6 59	I. 189°4073 72°4192 284°6759 161°6085	1°002238 0°996450 0°998948 1°004735	-2°04240 +1°40284 +0°58869 -8°18960	+0°0001 +0°616 -1°476 +1°648	+0°12 -5°09 +5°97 -2°38	+0°01464 +3°46186 +2°64825 -6°12476	-0°0308 -110 -56 +472	-131 91 9 49

Thin cloud about.

Cloudy.

Plate No. Obsr. Defn.	Date 1833. G.M.T.	Satellite.	η .	ρ .	α .	γ .	δ .	x_0 hs.	n .	Resid.	Remarks.
4129 JJ 3-4	Sept. 15 h m s 7 34 4	I.	67°43'19	1°00'14.72	+1°03'55.2	+0°04'90	-4°55	+1°21'939	-°00234.	+°00040	
		II.	10°06'73	1°00'47.77	+3°22'05.2	-°01'86	-0°18	+3°41'171	-	-	54
		III.	72°86'86	0°99'85.8	+2°19'05.8	+°13'90	-5°40	+2°38'038	-	+261	24
		IV.	224°79'98	0°99'69.52	-7°20'42.8	-°10'95	+3°11	-7°01'151	+636	-	10
4130 JJ 3-4	Sept. 17 h m s 8 51 7	I.	124°33'23	1°00'36.95	-0°09'37.3	+0°03'17	-4°72	+1°58'054	+°00475	-	64
		II.	219°01'83	0°99'50.8	-2°71'66.8	-°02'24	+2°92	-0°22'569	+197	+78	
		III.	175°87'64	1°00'22.76	-4°08'79.2	+°02'00	-1°28	-2°49'634	+256	-38	
		IV.	269°55'99	0°99'30.70	-1°34'77.5	-°24'35	+5°10	+1°14'150	+23	+22	
4131 JJ 2-3	Sept. 20 h m s 9 8 15	I.	18°09'77	0°99'85.57	+1°05'79.6	+0°00'82	-0°87	-0°60'960	-°00046	+96	
		II.	164°80'87	0°99'31.20	-2°88'01.4	+°04'97	-2°20	-5°44'278	+446	-67	
		III.	327°39'00	0°99'78.29	+3°79'48.5	-°08'62	+3°84	+1°22'856	+81	+48	
		IV.	335°43'53	0°99'47.49	+7°39'57.2	-°18'17	+2°79	+4°82'384	-478	-76	
4132 GD	Sept. 22 h m s 7 14 35	I.	49°00'31	1°00'06.50	+1°50'49.5	+0°03'54	-3°36	-2°53'407	-°00023	+39	
		II.	358°32'59	1°00'54.90	+3°11'06.6	-°03'39	+0°98	-0°92'765	+48	-150	
		III.	64°37'41	0°99'71.8	+2°81'89.2	+°12'52	-4°83	-1°21'929	+58	+98	
		IV.	17°12'82	0°99'52.1	+8°72'06.3	-°01'53	-0°77	+4°68'103	-81	+15	
4134 JJ 3-4	Sept. 24 h m s 9 25 8	I.	114°07'65	1°00'36.29	-0°52'43.0	+0°05'38	-4°98	+0°18'347	-°00211	+27	
		II.	211°87'13	0°99'91.25	-2°86'09.1	-°01'03	+2°15	-2°15'546	+367	+9	
		III.	169°26'29	1°00'22.44	-4°71'53.5	+°03'62	-1°93	-4°00'131	+416	-23	
		IV.	62°02'23	1°00'48.31	+5°26'91.6	+°16'67	-4°12	+5°97'329	-575	-14	

Hazy at times.

Very bad plate.

Thin cloud at times.

N. W. wind shaking in-
strument.

Thin cloud about.

Bad images.

Baxendell's Observations of Variable Stars. Sixth Instalment.
No. 13 (T Herculis), Revised; and No. 16, R Persei. Edited
by Professor H. H. Turner and Mary A. Blagg.

No. 13, T Herculis (6512 Chandler).

18^h 3^m 37^s + 30° 59'·9 (1855·0)

18 5 19 + 31 0·2 (1900·0)

[*Introductory Note by H. H. Turner.*—On the publication of the Fifth Instalment, Mr. C. L. Brook remarked that he could not agree with the identifications of some of the comparison stars. These had indeed puzzled the Editors, there being apparently some change about 1863; but as there was no independent record of change, the reductions were made with some allowance for apparent idiosyncrasies. Mr. Brook's criticism led to further search, and the matter was cleared up. The facts are these: on receipt of the Baxendell MSS., I gathered all the charts into a special book. Tracings were made for working use, and the book was put away in a safe place. A similar plan was adopted with the observing books; they were copied out and work done with the copies, reference being made to the originals when any question arose. As in course of time Miss Blagg gradually took over the whole of the reductions, the original books were ultimately sent to her to facilitate references; but unfortunately the book of diagrams, which had been stored separately, was not sent. The omission was discovered owing to Mr. Brook's remark, and a new chart of comparison stars for T Herculis was found, in which the same letters (from *a* to *k*) were used as in the first chart, but allotted to quite different stars (as below). The second chart is undated, but was clearly used from 1863 August 15. The stars in the first chart were correctly identified, but as these identifications did not apply to any observations after 1863 July, it has been deemed simplest to do the whole work for this star afresh. I tender humble apologies for the oversight.—H. H. T.]

TABLE I.

Baxendell's Comparison Stars.

Baxendell.				A.S.V. II.		Harvard.		Adopted Mag.	Baxendell's Estimate.
First Chart, 1858 Nov. 7.	Later Chart, 1863 ? Aug.	Sketch, 1863 May 23	Sketch, 1886 Nov. 30.	No.	Grade.	Letter.	Mag.		
<i>a</i>	...	..	...	2	...	...	...	...	...
<i>b</i>	...	.	...	1	...	...	..	...	..
<i>c</i>	...	.	...	3	0	<i>a</i>	6·67	...	...
<i>d</i>	<i>a</i>	...	...	4	11	<i>c</i>	7·17	73	7·3 or 7·5
<i>e</i>	<i>b</i>	...	...	6	19	<i>e</i>	7·82	78	7·8
<i>f</i>	<i>c</i>	...	...	5	18	<i>f</i>	8·10	79	7·7 ^v
<i>j</i>	<i>d</i>	...	...	11	39	...	(8·85)	88	8·8

TABLE I.—*Baxendell's Comparison Stars*—continued.

Baxendell.									
First Chart, 1858 ov. 7.	Later Chart, 1863 ? Aug.	Sketch, 1863 May 23.	Sketch, 1886 Nov. 30.	A.S.V. II.		Harvard.		Adopted Mag.	Baxendell's Estimate.
				No.	Grade.	Letter.	Mag.		
...	<i>e</i>	...	...	8	...	<i>h</i>	8.69	87	8.7 <i>v.</i>
<i>g</i>	<i>f</i>	...	...	14	47	<i>m</i>	9.44	94	9.6 <i>v.</i>
<i>h</i>	<i>g</i>	...	...	22	65	<i>n</i>	9.82	98	10.0
<i>k</i>	...	...	...	23	68	...	(10.33)	103	...
...	<i>h</i>	<i>x</i>	...	31	84	...	(11.16)	112	10.9 <i>v.</i>
...	<i>k</i>	<i>y</i>	<i>k</i>	29	79	<i>p</i>	10.97	110	10.7 or 10.9
...	<i>l</i>	...	...	34	...	<i>q</i>	11.65	117	11.8
...	<i>m</i>	...	...	45	...	<i>r</i>	12.41	122	12.2 or 12.3
...	comp. <i>k</i>	...	...	...	...	...	...	128	13.1

The Harvard magnitudes not in brackets are from *H.A.*, xxxvii. p. 172; those in brackets are interpolated from the table on p. 200. The adopted magnitudes are in most cases those of Harvard, with a few exceptions to suit the observed intervals better.

TABLE II.

Baxendell's Differences between his Comparison Stars.

(The unit is a step, converted in the last line into 0.1 mag.)

Years.	78-73.	79-78.	*79-73.	88-79.	88-87.	87-79.	88-78.	87-78.	94-87.	98-94.	112-98.	112-110.	117-112.	117-110.	122-117.
1858-9	3.0 ₁	+0.5 ₅	...	9.3 ₈	...	...	10.8 ₂	...	...	1.5 ₁	...	...	...	...	...
1863-5	4.0 ₁₆	-0.5 ₁₀	3.7 ₆	15.0 ₁	...	...	11.0 ₁	...	...	3.0 ₁	3.5 ₁	-1.0 ₁	...	...	5.0 ₁
1866-8	4.7 ₂₁	-0.6 ₉	3.9 ₄	12.2 ₂	2.0 ₁	5.7 ₃	10.0 ₆	6.9 ₈	9.2 ₆	3.0 ₄	10.0 ₂	+1.1 ₄	8.0 ₁	6.0 ₁	5.0 ₁
1878-80	5.2 ₁₄	-1.0 ₅	9.0 ₁	10.3 ₅	2.7 ₅	6.8 ₃	9.5 ₂	6.0 ₂	9.4 ₅	6.0 ₅	5.8 ₂	+2.9 ₁₃	4.2 ₅	8.1 ₄	4.0 ₁
1881-2	5.9 ₅	+3.0 ₂	...	...	...	10.0 ₁	...	9.0 ₂	12.0 ₁	6.5 ₂	...	-2.8 ₃	...	...	7.0 ₁
1883-6	7.8 ₈	+2.5 ₂	...	12.0 ₁	1.5 ₃	9.3 ₂	18.0 ₂	9.8 ₄	6.0 ₂	7.2 ₄	12.0 ₁	+2.5 ₂₃	5.8 ₂	7.8 ₃	6.0 ₁
Mean.	5.1	0.0	*4.3	11.0	2.2	7.3	11.3	7.7	9.0	5.3	7.8	+2.0	5.1	7.7	6.0
No. of Comparisons.	65	34	11	12	9	9	13	16	14	17	6	44	8	8	1
Reduced Mean.	4.5	0.0	*3.6	9.2	1.8	6.3	9.5	6.6	7.5	4.6	6.1	+1.7	4.2	6.6	5.0

Comparison with the table formerly given (p. 159) will show that the comparisons of 1858-9 are now brought much more nearly into agreement with the later ones, while no real difference is made in the means. The peculiar variations and fluctuations in the comparisons also remain as before, except that those between the observations of 1858-9 and those of other years are now of about the same character and amount as the rest.

For the telescopes used, see *M.N.*, lxxvi. p. 160.

* These comparisons nearly all occur while the star *c* was brighter than *b*.

TABLE III.

Baxendell's Observations of T Herculis.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1859				1864			
Aug. 1	0258	78+5, 88-3, 79+3	85	May 26	2018	122+6	128
2	0259	79+T=88-T	84	July 13	2066	73+5, 78+0	78
10	0267	79+T=88-T, 78+4	83	16	2069	73+4, 78-1	77
18	0275	79+2, 88-6	82	19	2072	73+1, 78-3	74
19	0276	78+3, 79+4, 94-5	84	29	2082	73-1, 78-4	73
24	0281	78+2	80	Aug. 21	2105	88-3	85
Sept. 4	0292	78+8, 79+8, 88-2	86	23	2107	88-3	85
10	0298	88+2	90	1865			
28	0316	?98-2 or 110-2 or 108	?96	May 1	2358	117+3, 122-1	120
1863				24	2381	98-1, 112-4	102
May 23	1649	110+14	124	June 9	2397	78+3	81
June 3	1660	Invisible<110		11	2399	73+4, 78+1, 79+1	79
12	1669	„ <120		13	2401	73+2, 78+1, 79-1	77
20	1677	„ <127		17	2405	73+3, 78-1	76
25	1682	„ <125		20	2408	73+3, 78-2	76
July 3	1690	„ <125		28	2416	73+3, 78-1	76
Aug. 15	1733	73+2, 78-1, 79+0	77	July 1	2419	73+2, 78-1	76
20	1738	73-2, 78-3	73	11	2429	73+4, 78+1, 79+1	79
24	1742	73-2	71	21	2439	88+0	88
Sept. 5	1754	73-2	71	Sept. 7	2487	Not seen.	
19	1768	73+5, 78+1	78	Oct. 7	2517	122+3	125
Oct. 9	1788	88+8, 94-3	94	29	2539	98-2	96
Nov. 2	1812	122+0	122	Nov. 12	2553	*? 78+6	? 84
29	1839	Glimpsed with difficulty, ? +3	? 125	15	2556	78+3, 88-6	82
Dec. 17	1857	Not seen<125		26	2567	79+1, 78-1	78
1864				1866			
Apr. 25	1987	„ <110		Apr. 23	2715	78+4, 88-3	84
May 7	1999	Glimpsed 132		25	2717	78+3, 88-4	82
				29	2721	78+3, 88-6	82
				May 7	2729	78+1, 88-7	80
				15	2737	73+2, 78-1, 88-9	77

* The query is B.'s.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deducted Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deducted Mag.
1866				1867			
May 17	2739	73+3, 78-1	76	May 11	3098	87+2, 94-5	89
20	2742	73+3, 78-1	76	14	3101	87+5, 94-2	92
22	2744	73+3, 78-1	76	21	3108	87+7, 94-1	94
26	2748	73+3, 78-1	76	July 24	3172	Glimpsed with difficulty	
June 10	2763	78+2, 88-7,	81	27	3175	128+2	130
		79+2		Aug. 1	3180	128+0	128
25	2778	88+4, 94-3	92	6	3185	128+0	128
July 31	2814	122+0	122	21	3200	110+3, 117-2	114
Aug. 27	2841	122+1	123	Sept. 2	3212	94+1, 98-2	96
31	2845	122+1	123	5	3215	87+9, 94-2	94
Sept. 8	2853	117+3, 122-3	120	10	3220	87+2, 94-9	87
15	2860	112+1, 110+3	113	16	3226	78+5, 87-3	84
17	2862	110-1, 112-1	110	24	3234	78+4, 87-3	83
24	2869	94-1	93	Oct. 3	3243	78+1, 73+4	78
Oct. 1	2876	87-1, 94-4	88	23	3263	73+6, 78+2	80
6	2881	78+4, 87-3,	83	Nov. 1	3272	87+3	90
		79+3		19	3290	110-4	106
8	2883	78+1, 87-3	82	1868			
10	2885	79+0, 78-1,	80	Apr. 11	3434	78+2, 87-3	82
		87-3		May 11	3464	110+2	112
14	2889	73+3, 78-1,	78	30	3483	122+1	123
		79+1		June 4	3488	122+3	125
19	2894	73+0, 79-3,	75	12	3496	122+7	129
		78-3		15	3499	122+7	129
28	2903	73-1, 78-5	72	30	3514	122+2	124
Nov. 3	2909	73+0, 78-3	74	July 11	3525	117-2	115
6	2912	73+0, 78-4	74	12	3526	112+3, 117-3	114
11	2917	73+0, 78-5	73	21	3535	98+3, 112-7	103
19	2925	73+0, 78-3	74	Sept. 4	3580	73+3, 78-1	76
25	2931	73+5, 78+1,	80	6	3582	73+3, 78-1	76
		87-5		11	3587	73+6, 78+1	79
30	2936	78+2, 87-5	81	Oct. 17	3623	110+1	111
Dec. 16	2951	94+2, 98-2,	99	Nov. 8	3645	122-2	120
		112-8		15	3652	117+2, 122-3	119
1867				1869			
Apr. 11	3068	73+4, 78+1,	79	Apr. 30	3818	122+6	128
		79+1		June 8	3857	117+0	117
21	3078	73+4, 78+1,	78				
		79+0					

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1869				1879			
Oct. 26	3997	Not seen < 133		Aug. 11	7573	117+0, 112+3	116
1878				14	7576	112+6, 122-2, 117+2	119
June 1	7137	117+0	117	23	7585	122+3	125
10	7146	117-5, 112-2	111	30	7592	122+5	127
23	7159	94+3, 98-3	96	Sept. 2	7595	122+6	128
28	7164	87+2, 94-3	90	7	7600	122+7, min. vis.	
July 8	7174	79+3, 88-5	82			-3	129
19	7185	73+9, 79+3, 78+1	81	13	7606	122+3	125
27	7193	78+1, 79+2	80	19	7612	122+3	125
31	7197	78+3	81	23	7616	122+0	122
Aug. 6	7203	78+3, 88-4, 79+6	83	25	7618	122-2	120
8	7205	79+5, 88-4, 87-1	85	29	7622	122-3	119
18	7215	87+7, 94-2, 88+2	92	Oct. 4	7627	112+1, 117-3, 110+5, 122-7	114
21	7218	87+5, 94-3	92	15	7638	98+0	98
25	7222	94+0, 98-4	94	Nov. 1	7655	87+3, 94-3	90
28	7225	94+3, 98-1	97	5	7659	87+2	89
Sept. 2	7230	98+0, 110-3	98	13	7667	79+3, 87-3	83
9	7237	110-1	109	23	7677	73+3, 78-2	76
11	7239	112+0, 110+3	112	25	7679	73+3, 78-3	76
13	7241	110+2, 112-1	112	27	7681	73+3, 78-1	76
24	7252	122-2	120	29	7683	73+3, 78-2	76
Oct. 13	7271	122+6	128	Dec. 3	7687	73+2, 78-2	76
17	7275	122+7	129	1880			
26	7284	122+3	125	Mar. 17	7792	98+0	98
Nov. 1	7290	122-1	121	Apr. 1	7807	94-1	93
4	7293	122-2	120	11	7817	87-1	86
8	7297	112+0	112	24	7830	78+1, 87-3	82
28	7317	87+5, 94-7	90	30	7836	73+3, 78-2	76
Dec. 8	7327	78+3, 87-4, 88-6, 79+2	82	May 6	7842	78+2	80
1879				9	7845	73+4, 78+0	78
Mar. 31	7440	122+6	128	13	7849	73+4, 78+0	78
Aug. 8	7570	112+2, 117-2	114	16	7852	73+2, 78-1	76
				20	7856	73+4, 78+3	79
				24	7860	73+5, 78+2	79
				28	7864	73+5, 78+2	79
				June 3	7870	87-1	86

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240 + .	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240 + .	Corrected Comparisons.	Deduced Mag.
1880				1881			
June 17	7884	94 + 1, 98 - 5	94	Aug. 1	8294	110 - 2	108
July 4	7901	117 - 2	115	24	8317	78 + 6, 87 - 2	84
20	7917	122 + 3	125	31	8324	78 + 1	79
30	7927	122 + 1	123	Sept. 14	8338	73 + 4, 78 - 3	76
Aug. 9	7937	122 - 1	121	19	8343	73 + 3, 78 - 3	76
11	7939	122 - 1	121	Oct. 2	8356	87 - 3	84
13	7941	122 - 2	120	15	8369	98 - 3	95
24	7952	112 - 3, 110 - 2,	106	20	8374	*88 + 4, *87 - 3	88
		98 + 3					or 95
29	7957	98 - 1	97	29	8383	112 + 2	114
Sept. 4	7963	98 - 3, 94 + 3	96	Nov. 17	8402	112 + 4	126
15	7974	88 + 1, 87 + 0	88	Dec. 1	8416	122 + 6	128
26	7985	73 + 1, 78 - 3	74	19	8434	122 + 3	125
28	7987	73 + 0	73				
{ Oct. 2	7991	73 + 0	73	1882			
9	7998	73 - 2	71	Mar. 18	8523	79 + 3, 98 - 10,	86
10	7999	73 - 1	72			78 + 9, 94 - 5	
20	8009	73 - 2	71	Apr. 6	8542	98 + 1	99
29	8018	73 + 2, 78 - 1	76	10	8546	112 - 2	110
Nov. 3	8023	78 + 2	80	20	8556	112 + 5, 110 + 1	114
21	8041	78 + 9, 94 - 2	90	May 12	8578	122 + 4	126
26	8046	94 + 3, 98 - 4	96	14	8580	122 + 3	125
Dec. 1	8051	98 - 1, 110 - 6	100	20	8586	122 + 6	128
				28	8594	122 + 10	132
1881				July 9	8636	94 + 0, 98 - 5	94
Mar. 28	8168	73 - 2	71	24	8651	73 + 5, 78 + 0	78
Apr. 1	8172	73 + 1, 78 - 2	75	Aug. 19	8677	78 + 2	80
17	8188	73 + 4, 78 + 0	78	Sept. 4	8693	78 + 9, 87 + 3	88
May 4	8205	98 - 1	97	9	8698	87 + 9, 94 + 0	85
7	8208	98 - 2	96				
16	8217	112 + 0, 110 - 1	110	1883			
20	8221	112 + 3, 110 + 0	112	Mar. 24	8894	117 + 4, 122 - 2	120
29	8230	122 + 1	123	Apr. 8	8909	<122 + 4	<126
June 12	8244	122 + 3	125	July 28	9020	94 - 3	91
July 8	8270	122 + 4	126	Aug. 25	9048	117 + 2, 122 - 3	119
17	8279	117 + 0, 122 - 6	116	Oct. 2	9086	122 + 6	128
27	8289	110 - 1	109	28	9112	112 + 2	114

* ? Error for 91 and 98.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1884				1884			
Mar. 21	9257	122+3	125	Nov. 19	9500	73-2	71
25	9261	122+3	125	28	9509	73+4	77
Apr. 7	9274	117+1, 122-7	116	Dec. 8	9519	87+7	94
12	9279	110+2, 117-7	111	20	9531	94+4, 98-3	96
May 4	9301	87-1	86	1885			
18	9315	78-2, 73+5	77	Apr. 8	9640	79+6, 88-4,	85
22	9319	73+5, 78-2	77			87-2	
24	9321	73+5, 78-2	77	17	9649	78+5, 79+1	82
26	9323	73+3, 78-3	76	19	9651	79+3, 87-5,	82
30	9327	73+1, 78-5	74			78+3	
June 4	9332	73+5, 78-3	76	23	9655	78+1	79
11	9339	78+3	81	26	9658	78+3, 87-4	82
20	9348	87-3, 78+10	86	May 7	9669	78+5	83
25	9353	78+8, 87+4,	92	10	9672	78+3	81
		94+2, 98-4		16	9678	87-2	85
July 2	9360	94+1	95	June 1	9694	98-2, 112-6	101
12	9370	98+4, 110-3	104	9	9702	110+1, 112-1	111
27	9385	*112+7, *98-3	?107	July 4	9727	122+4	126
Aug. 1	9390	117+0	117	25	9748	Invisible <122+2	
6	9395	117+0 or fainter	?117	29	9752	122+5 (only	127
11	9400	122+0	122			glimpsed)	
14	9403	122+0	122	Aug. 3	9757	122+5 min. vis.	127
19	9408	122+4	126			-5	
23	9412	122+4	126	14	9768	122+0	122
25	9414	122+4	126	20	9774	117+1, 122-4	118
Sept. 1	9421	122+2	124	28	9782	110+5, 112+0	114
5	9425	About 122+0	?122	Sept. 3	9788	110+3, 112-3	111
13	9433	112+2	114	13	9798	94+1, 98-3	95
18	9438	112+1	113	25	9810	73+9, 78+1	80
20	9440	110+5, 112-1	113	27	9812	78+0	78
Oct. 5	9455	88+1, 87-1	88	Oct. 29	9844	73+6, 78+1	79
12	9462	78+6, 88-7	82	Nov. 8	9854	87+4, 94-3	91
Nov. 7	9488	73-4	69	15	9861	94-2	92
10	9491	73-4	69	19	9865	94+2, 98-5	94
13	9494	73-2	71	Dec. 1	9877	110+1, 112-1	111
17	9498	73-2	71	26	9902	122+3	125

* ? Designations accidentally transposed.

TABLE III.—*Baxendell's Observations of T Herculis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1886	241+			1886			
Apr. 6	0003	78+2	80	Nov. 30	0241	122+6	128
26	0023	87-1	86	Dec. 4	0245	122+6	128
May 21	0048	110+2, 117-1	114	25	0266	122+0	122
Sept. 14	0164	78-3	75	29	0270	122-4	118
Oct. 23	0203	110-3	107				
Nov. 2	0213	110+8, 117+0	118	1887			
16	0227	122+3	125	Jan. 12	0284	98-3	95

The effect of correcting the erroneous identifications previously made has been to make the deduced magnitudes come out, roughly speaking, a magnitude fainter throughout,* except in the years 1858-9, when the first chart was used. These early observations are no longer irreconcilable with the rest, but merely suggest a rather less bright maximum than usual.

These corrected results have been arranged and tabulated as before, with assumed period = 165 days. Table IV. shows the means for the twelve portions for the same groups of periods as before. Comparison with the similar table on page 165 shows that the apparent changes of epoch have not been at all affected by the corrections. As before, Group B shows an advance of about one portion on Group A, and Groups C, D, and E show a retrogression of about two portions on group B. The mean is obtained, as before, by shifting the portions accordingly.

TABLE IV.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A (X. to XVIII.)	124 ₄	111'5 ₁	92'5 ₄	79 ₆	75'5 ₇	75 ₅	83 ₈	93 ₃	99 ₁	123 ₂	122 ₁	121 ₂
B (XIX. to XXIII.)	129 ₁	121 ₃	108'5 ₂	92 ₁	83 ₁	77 ₂	79 ₂	86 ₂	108'5 ₂	112 ₁	121'5 ₂	121 ₂
C (XLIII. to XLIX.)	106 ₅	94'5 ₄	84 ₅	76'5 ₅	78 ₅	82'5 ₂	95 ₄	109 ₂	118 ₄	125 ₃	126 ₅	116 ₆
D (L. to LVI.)	111 ₃	90 ₂	79 ₃	76 ₂	78 ₃	88 ₅	99 ₄	116 ₃	120 ₃	126 ₃	127 ₅	111 ₆
E (LVII. to LXII.)	116 ₃	97 ₃	86'5 ₂	76 ₃	77 ₄	83 ₃	94 ₄	110 ₃	119 ₃	125 ₂	127'5 ₂	124 ₄
Mean, with shifted positions.	120 ₁₄	110 ₁₄	94 ₁₄	82 ₁₇	76 ₁₉	77 ₁₉	85 ₁₈	97 ₁₇	111 ₁₀	120 ₁₄	125 ₁₂	121 ₁₂

Each observation has then been compared with the curve thus formed, and Table V. gives the mean result, for each period, of each portion. As the differences would have been too large to be trustworthy, had the observations been retained in their original position, they have been shifted in the same manner as was done in forming the mean curve, before comparing them with it. There is thus a general correction of -14 days to be applied to the periods in B, and one of +14 days to those in C, D, and E. The place of maximum, as read from the curve, is 68 days after the point which corresponds with the initial date of each period. In obtaining corrections to epoch, the

* Less at maximum and more at minimum.

observations in square brackets have not been used, and some only of those in round brackets, the unused ones being too near maximum or minimum to be of any assistance; and the lower figures for $p-f$ have been taken in portions 5, 6, 11, and 12, as being more correct for the parts of those portions which alone were used.

TABLE V.
Comparison of the Observations with a Mean Light Curve.
(The unit is 0.1 magnitude.)

Period.	Initial Date 240 ±.	Reference Numbers and Mean Magnitudes of Portions.											
		(1.) 120.	(2.) 110.	(3.) 94.	(4.) 82.	(5.) 76.	(6.) 77.	(7.) 85.	(8.) 97.	(9.) 111.	(10.) 120.	(11.) 126.	(12.) 127.
I.	0200	...	...	...	...	(+8)	(+4)	-1	-2	?-3	...	...	...
IX.	1520	...	...	...	...	...	...	...	...	...	+5	...	?-2
X.	1685	...	...	...	-6	-7	[-4]	-3	-3	...	+4	...	...
XII.	2015	+6	...	...	-2.5	[-3]	...	-1	...	...	...	...	...
XIV.	2345	+5	...	+9	+0.5	[0]	[+1]	-1	...	...	...	...	...
XV.	2510	+5	...	-5	-3	+1	...	...	...	...	...	...	...
XVI.	2675	...	...	-6	-2	[+1]	[+1]	-3	-5	...	...	-1	...
XVII.	2840	+1.5	+1	-7	-5	[-3]	[-2]	-8	...	-6	...	...	...
XVIII.	3005	...	...	...	...	[+3]	[+2]	+2	-3	...	...	...	[+2]
XIX.	3170 + 14	[+5]	0	-7	-1	[+1]	+1	+6	+6	...	...	...	...
XX.	3335	...	...	...	...	...	...	0	...	+3	+2	[+2]	...
XXI.	3500	...	-1	-7	...	...	[+1]	[+3]	...	+8	...	0	-4
XXII.	3665	...	...	...	...	...	...	...	...	...	...	+5	...
XXIII.	3830	...	+1	...	...	...	...	...	...	...	...	...	...
LIII.	7130 - 14	...	+7	+10	+8	+5	(+4)	0	-4	-6	-4	...	[+1]
LIV.	7295	...	+3	-2	-5	-2	...	...	...	...	...	...	[+1]
XLV.	7460	...	...	...	...	...	...	...	...	...	-1	[+1]	[0]
LVI.	7625	...	+1	-3	...	+5.5	(+2)	[0]	...	...	...	...	...
LVII.	7790	...	...	-16	-5	+3	(+2)	(+1)	-3	-8	...	-2	[-2] (-4.5)
LVIII.	7955	...	-9	-15	-9.5	-11	(-5)	-0.5	...	-3	...	...	...
LIX.	8120	...	...	...	...	[-2]	-3	...	+2	+4.5	+7	+2	[+1]
L.	8285	...	-3	-2	...	0	[+1]	[0]	+2	+3	+8	+6	+3
LI.	8450	...	...	...	...	...	...	+3	+2.5	0	...	(+1)	(+5)
LII.	8615	...	...	...	-1	-3	...	[+3]	+2	+3	...	...	...
LIII.	8780	...	...	...	...	...	...	...	...	...	+2	...	...
LIV.	8945	...	...	...	...	...	...	+7	...	+8	...	...	[+1]
LV.	9110	...	...	0	...	...	...	...	...	...	...	...	0
LVI.	9275	...	0	-2	-4	-2	[-0.5]	(+2)	+4	+2	...	-2	(-1) (-2)
LVII.	9440	...	-5	-2	-13	...	[-6]	[-5]	+1	-2	...	...	...
LVIII.	9605	...	...	...	...	+3	[+5]	+3.5	+2	+4	+6	+4	...
LIX.	9770	...	+4.5	+7	+11	+4	0	...	-5	-5.5	-3	...	-1
LX.	9935	...	...	...	...	...	0	...	-9	...	-4	...	...
	241 +	...	...	...	...	...	...	...	...	...	...	...	...
LXI.	0100	...	...	...	...	...	-3	...	...	-4	0	+1	[+1]
LXII.	0265	...	...	+7	-2	...	...	...	...	...	...	...	...
$p-f$		+17	+26	+28	+18	(+5)	(-9)	-20	-26	-23	-15	(-7)	(+6)
						+12	-16					-12	+14

As already said, the observations of Period I. now fall into line with the rest; and the figures for the other periods do not differ much from the old amounts (pp. 166-7). Many of them are slightly increased, as we should expect, $p-f$ being generally larger. The *general* difference in brightness between one period and another has been taken into account in the working.

Table VI. gives the corrections to the assumed epoch, as obtained from the new figures.

TABLE VI.
Corrections to Assumed Epoch.

Period.	Assumed Epoch.	Correc- tion.	Period.	Assumed Epoch.	Correc- tion.
I.	0268 ^o 0	+6 ^o 1	XLIII.	7198 ^o 0	-8 ^o 3
X.	1753 ^o 0	-2 ^o 3	XLVII.	7858 ^o 0	-15 ^o 7
XII.	2083 ^o 0	-1 ^o 1	XLVIII.	8023 ^o 0	-19 ^o 4
XIV.	2413 ^o 0	+3 ^o 8	L.	8353 ^o 0	-17 ^o 7
XVI.	2743 ^o 0	-0 ^o 5	LII.	8683 ^o 0	-16 ^o 5
XVII.	2908 ^o 0	+1 ^o 7	LVI.	9343 ^o 0	-16 ^o 9
XIX.	3288 ^o 0	+9 ^o 0	LVII.	9508 ^o 0	-18 ^o 2
XXI.	3568 ^o 0	+9 ^o 7	LVIII.	9673 ^o 0	-15 ^o 1
			LIX.	9838 ^o 0	-8 ^o 3
			LX. to LXII.	0168 ^o 0	-9 ^o 6

Comparing these figures with those previously given (p. 167), we see that there is remarkably little difference—in two cases only does it amount to as much as two days,—and the new figures fit the formula found for the old ones better than the old ones themselves did! However, a slightly revised formula is found to fit both old and new results even better still. Table VII. gives the comparison of the new figures (Baxendell) with the old and new formulæ and also with Chandler, whose formula is:

$$2403399 + 165^{\circ}0 E + 10 \sin (5^{\circ} E + 110^{\circ}).$$

The formula previously found (partly by trial and error) was:

$$2400116^{\circ}4 + 164^{\circ}6 E' + 13 \sin (8^{\circ}5 E' + 211^{\circ});$$

and the revised one now suggested is:

$$2400117^{\circ}8 + 164^{\circ}6 E' + 14 \sin (8^{\circ}5 E' + 200^{\circ})$$

(where approximately $E' = E + 20$).

TABLE VII.
Comparison of Epochs of Maxima.

Period in E.	Baxendell.	Calculated 1st Formula.	B-C (1).	Calculated 2nd Formula.	B-C (2).	Chandler.	Bax.-Ch.
I.	0274.1	0272.7	+1.4	0275.7	-1.6	0266.6	+7.5
X.	1750.7	1750.7	0.0	1750.4	+0.3	1757.7	-7.0
XII.	2081.9	2082.1	-0.2	2081.2	+0.7	2088.4	-6.5
XIV.	2416.8	2414.3	+2.5	2413.1	+3.7	2418.8	-2.0
XVI.	2742.5	2747.1	-4.6	2745.9	-3.4	2749.0	-6.5
XVII.	2909.7	2913.6	-3.9	2912.5	-2.8	2914.0	-4.3
XIX.	3247.0	3246.6	+0.4	3245.8	+1.2	3243.7	+3.3
XXI.	3577.7	3579.4	-1.7	3579.0	-1.3	3573.1	+4.6
XLIII.	7189.7	7186.5	+3.2	7190.0	-0.3	7186.9	+2.8
XLVII.	7842.3	7840.3	+2.0	7842.5	-0.2	7844.9	-2.6
XLVIII.	8003.6	8004.4	-0.8	8006.1	-2.5	8009.6	-6.0
L.	8335.3	8333.5	+1.8	8334.4	+0.9	8339.2	-3.9
LII.	8666.5	8663.6	+2.9	8663.8	+2.7	8669.0	-2.5
LVI.	9326.1	9326.9	-0.8	9326.3	-0.2	9329.6	-3.5
LVII.	9489.8	9493.2	-3.4	9492.5	-2.7	9494.9	-5.1
LVIII.	9657.9	9659.6	-1.7	9658.8	-0.9	9660.3	-2.4
LIX.	9829.7	9826.1	+3.6	9825.4	+4.3	9825.8	+3.9
LXI.	0158.4	0159.1	-0.7	0158.6	-0.2	0156.9	+1.5

[There is a misprint in the old Table VII. (p. 168), for Period XVII., Bax.-Ch., should be -5.7, not +5.7.] Consideration of these results shows that it is not possible to fix definitely on one of these from Baxendell's observations alone.

Both in B-C (1) and in B-C (2) the "errors" are small and well distributed, and though the "fit" is best in C (2) it is not overwhelmingly so. The differences in Bax.-Chandler are never very large, but are clearly systematic. Inspection of the observations shows that the epochs in "Baxendell," C (1) and C (2), are all compatible with the observed magnitudes, except possibly in Period LVIII. (but the brightness seems to fluctuate a good deal near maximum in that period); while in several cases the Chandler dates are not compatible with these observations, though never very far out.

We come now to the question whether the undoubted variations in epoch are best represented by a continuous formula such as Chandler's, or by a discontinuous one. In discussing R Leonis (p. 184 of this volume) it was shown that the hypothesis of discontinuous changes in the period fitted the observations well; and it was remarked that the same was true of S Herculis. It seems possible that the same may be true of other stars, and even of all similar stars: for it may be fairly asserted that in no case has a

continuous change of the above type been proved to the hilt as yet.

Now Table VI. certainly suggests discontinuities. Between Periods LVIII. and LIX. there is a quite sudden jump of some 8 days: Periods XLVII. to LVIII. are steadily consistent, the mean correction being $-17^{\text{d}}.1$: then LIX., confirmed by LX. to LXII., changes to $-9^{\text{d}}.0$. There is an equal jump in the opposite direction somewhere between XLIII. and XLVII.: and the pair of opposite jumps are repeated about XVIII. (for the increase) and VI. (?). These are discontinuities of a different character from those noted for R Leonis and S Herculis, where the *period* suddenly altered without momentary change of *phase*. Here the *phase* alters: whether the period also alters is uncertain, owing to the long gap between XXI. and XLIII. We will return to the point in a moment: but first it may be remarked that a sudden change in *phase* seems to be exhibited also by Pogson's observations of R Ursæ Majoris (see *Mem. R.A.S.*, lviii. p. xix). In discussing them an attempt was made to fit them to Chandler's formula $11 \sin (8^{\circ} E + 238^{\circ})$, but it is remarked that the residuals "certainly indicate some kind of oscillation, but it is far from being clear as yet what is the law of oscillation." At the time of that discussion the hypothesis of possible discontinuities had not been seriously formulated. But, in view of other evidence, it now seems desirable to give the following explanation of the Pogson results. In the second line Pogson's phase is quoted from p. xix of the memoir above cited. In the next line is a simple correction for period: and the corrected phases then show a sudden jump of 41° between $E = 12$ and $E = 17$.

Pogson's R Ursæ Majoris.

Value of E	2	7	12	17	22	27	32
Pogson's phase	64°	67°	82°	133°	147°	157°	153°
Correction for period	0	-10	-20	-30	-40	-50	-60
Corrected phase	64	57	62	103	107	107	93
	61			102			

It will be noted that the period is *not* altered, only the phase, whereas in the case of R Leonis and S Herculis the period altered without the phase. No particular reason suggests itself why both period and phase should not be altered, when there is a real discontinuity: but it is worth paying attention to such facts as we have, however few; and thus it seems desirable to regard T Herculis as possibly like R Ursæ Majoris, where phase alters without alteration of period. To get at the period, we must look at a set of residuals where there is no discontinuity, such as those from XLVII. to LVIII., which are fairly steady, and show therefore that the adopted 165 days is not far from the true period. But there is a marked change in the unobserved gap between XIX.

(with correction $+9^{\text{d}}.0$) and XLIII. (correction $-8^{\text{d}}.3$). If we maintain the period unchanged, as above suggested, we must explain this by two drops of $8^{\text{d}}.6$, assuming the changes of phase to be all of the same amount, or by a single drop of twice the amount. In favour of the latter supposition is the fact that we could then separate the discontinuities by equal intervals of 13 periods, thus:—

VI.	XIX.	XXXII.	XLV.	LVIII.
$-8^{\text{d}}.4$	$+8^{\text{d}}.4$	$-16^{\text{d}}.8$	$-8^{\text{d}}.4$	$+8^{\text{d}}.4$

whereas we cannot have both equal intervals and jumps of the same amount, for it would be necessary to reduce the interval between jumps to 10 periods, which is certainly exceeded by XLVII. to LVIII. But to settle such points we must clearly have recourse to other observations: Baxendell's alone leave too many possible alternatives, including that of a change of period along with the change in phase.

There is an excellent series of observations of T Herculis by the Variable Star Section of the British Astronomical Association (appendix to sixth and seventh reports of the section) in the years 1900–1909. The maxima printed in the text are read as the highest points of the “most probable light curves.” But we can get more consistent results by reading off twelve points on these curves at equal intervals, and treating them in the way adopted for Baxendell's results. A series of dates was chosen following those adopted in Table V. in regular sequence, and the following corrections to epoch, strictly comparable with those of Table VI., were obtained. The B.A.A. readings are added for comparison.

Period.	Assumed Epoch 241+.	Calculated Correction.	B.A.A.	Period.	Assumed Epoch.	Calculated Correction.	B.A.A.
		d	d			d	d
XCII.	5283.0	+11.8	+4	CVI.	7593.0	+12.0	+16
XCIV.	5613.0	+2.6	+2	CVII.	7758.0	+5.4	+2
XCVI.	5943.0	+0.7	+1	CVIII.	7923.0	+6.1	-4
CI.	6768.0	+3.0	0	CIX.	8088.0	-2.5	-9
CIII.	7098.0	+5.8	+2	CX.	8253.0	+6.1	+3
CIV.	7263.0	+2.5	-1	CXI.	8418.0	+7.6	+13
CV.	7428.0	+3.5	+6	CXII.	8583.0	+1.3	+5

Now it is somewhat difficult to identify any discontinuities in the observations of the type shown by Baxendell's. There is a possible drop of the kind required somewhere between XCII. and XCIV.: but the epoch for XCII. depends on comparatively few observations near maximum. Let us, however, accept it by way of illustration. It follows LVIII. by about 35 periods—36 at most. This is not inconsistent with a discontinuity at about every 13 periods: but in that case it must be followed by another at about XCIII. + XIII. = CVI., and again at CVI. + XIII. = CXIX.

The latter epoch is not yet reached, but the evidence for a change at the former is not good. We have

	Calc.	B.A.
Epochs XCIV.-CV.	+2 ^s ·8	+1 ^s ·7
„ CVI.-CXII.	+5 ^s ·1	+3 ^s ·7
„ excluding CIX.	+6 ^s ·2	+5 ^s ·8

Even if we leave out the exceptional CIX. we still cannot get a change of 8·4 days, or half of it. The hypothesis which suits the B.A.A. observations best is a constant period, the mean correction to adopted epoch being +4^s·1 days. From this Periods XCII., CVI., and CIX. show the sensible deviations +7^d·7, +7^d·9, and -6^d·6, but none of them is supported by adjacent periods. The mean B.A.A. curve agrees closely with Baxendell on the rise and fall, but is a few tenths fainter near both maximum and minimum. See further note at end of paper.

The results of analysing each group of Baxendell's periods (except B, which contains very few observations) are given below, and compared with the figures for the mean curve:—

Mean Curve: $2\cdot97 \cos(\theta - 313^\circ) + 0\cdot14 \cos 2(\theta - 23^\circ) + 0\cdot04 \cos 3(\theta - 9^\circ)$

Group A: $3\cdot11 \cos(\theta - 319^\circ) + 0\cdot25 \cos 2(\theta + 3^\circ) + 0\cdot24 \cos 3(\theta - 28^\circ)$

„ C: $2\cdot90 \cos(\theta - 284^\circ) + 0\cdot06 \cos 2(\theta - 68^\circ) + 0\cdot06 \cos 3(\theta - 60^\circ)$

„ D: $2\cdot99 \cos(\theta - 276^\circ) + 0\cdot08 \cos 2(\theta - 173^\circ) + 0\cdot10 \cos 3(\theta - 108^\circ)$

„ E: $2\cdot98 \cos(\theta - 291^\circ) + 0\cdot21 \cos 2(\theta - 181^\circ) + 0\cdot13 \cos 3(\theta - 102^\circ)$

The changes in phase in the principal term are in very close agreement with the corrections to epoch. These analyses have, of course, been worked out entirely afresh, from the new figures, without any reference to the old results; but when we compare them with the old results (p. 169) we find that the agreement is so close as to be quite remarkable. In the first term in each expression the identity is almost exact, and in the other terms the difference is generally quite small, in spite of the fact that these terms almost always come out irregularly, being very easily affected by a slight want of smoothness in the curve.

This result of the new work is very satisfactory and encouraging, as it shows that even the wrong identification of *every* comparison star had imported practically no error into either the character of the mean light curve or the nature of the variations of period and epoch. It had merely increased the general brightness and lessened the range of the variation. This result is so pleasing that it is worth the price of the extra time and trouble involved, and almost worth the price of having made the mistake. This would not have been the case, of course, had the true magnitudes of the supposed comparison stars been taken; but fortunately the magnitudes were largely modified to suit Baxendell's intervals. It seems clear that, when any doubt exists as to the identity of a comparison star, it is better to assume for it a magnitude suited to the observed intervals, rather than to fix upon one particular star with which to identify it.

No. 16, *R Persei* (1222, Series III., Chandler; 032335 Harvard).

$$\begin{array}{l} 3^{\text{h}} 20^{\text{m}} 50^{\text{s}} + 35^{\circ} 10'.1 \text{ (1855.0)} \\ 3 \quad 23 \quad 41 + 35 \quad 19.6 \text{ (1900.0)} \end{array}$$

Identification of Comparison Stars.

There is one chart, and one small sketch, in the latter of which two stars only, besides R, are named. The stars on the chart are in some cases quite easily identified, and their Harvard magnitudes suit the observed intervals fairly well. But there is a difficulty in regard to the star ϵ , which is clearly (as shown on the chart) A.S.V. 36, the Harvard magnitude of which is 11.97. But this magnitude is entirely incompatible with the observed intervals between it and other stars, and it seems highly probable that the ϵ of the observations was A.S.V. 31 (Harvard mag. 11.34), which was not marked on the chart, or else that it was the ϵ' of the chart (A.S.V. 34, Harvard mag. 11.72), which was used only once under its own name. As its identity is so doubtful, its magnitude is assumed as 11.5, which best suits the observed intervals. Baxendell's e is not marked on his chart, but an identification of it with A.S.V. 23 fits the observed intervals well enough, except in the years 1865-6, when it appears as if either e was abnormally bright, or a different star was observed under that name. These comparisons have therefore not been used.

TABLE VIII.

Comparison Stars.

Baxendell.									
Chart.	Sketch, 1881, Feb. 28.	Other Names.	Estimates.		A.S.V.		Harvard.		Adopted Mag.
			Early.	Later.	No.	Grade.	Letter.	Mag.	
<i>a</i>	...	...	7.5	6.7	2	0	<i>a</i>	7.06	72
<i>b</i>	...	...	8.8	8.5	5	41	<i>d</i>	9.00	90
<i>c</i>	...	...	9.1	9.2	9	53	<i>e</i>	9.52	96
<i>d</i>	...	...	9.3	9.6 <i>v.</i>	17	67	<i>g</i>	10.10	101
<i>d'</i>	...	...	9.4	9.9	21	73	<i>h</i>	10.36	104
...	...	<i>e</i>	9.7	10.5	23	82	<i>k</i>	10.84	108
<i>e'</i>	...	...	...	...	34	105	<i>m</i>	11.72	117
ϵ	...	...	11.0	...	[36	118	<i>p</i>	11.97]	115
..	<i>f</i>	...	11.8	12.7	40	128	<i>q</i>	12.27	123
..	<i>f'</i>	...	...	- <i>f</i>	...	...	...	...	...
..	...	Star pre- ceding }	13.1	...	58	173	<i>u</i>	13.16	132

TABLE IX.

Baxendell's Differences between his Comparison Stars.

Years.	90-91.	96-90.	101-96.	104-101.	108-104.	123-108.	108-101.	123-104.	104-96.	123-115.
1861-4	15'0 ₁	7'5 ₁₄	6'8 ₁₂	1'0 ₁	...	15'0 ₁	6'7 ₃	...	...	9'0 ₃
1865-9	24'0 ₁	7'0 ₇	7'7 ₆	3'9 ₈	[1'3 ₇]	[14'5 ₂]	[0'8 ₆]	20'2 ₅	6'0 ₁	6'0 ₁
1878-83	16'1 ₄	4'9 ₁₁	4'3 ₂	1'8 ₂	3'5 ₂	18'8 ₄	8'8 ₂	...	7'1 ₄	...
1884-7	20'0 ₈	7'7 ₁₃	6'1 ₃	4'2 ₃	4'0 ₃	...	8'5 ₁	21'8 ₂	11'0 ₂	10'3 ₃
Means	19'1	6'8	6'6	3'4	3'8	18'0	7'7	20'7	8'1	9'1
No. of Observations	14	45	23	14	5	5	6	7	7	7
Reduced Means	14'7	5'8	4'7	3'0	3'3	14'2	6'6	16'4	6'6	7'6

The observations are too few, except in the first four cases, to be of much value.

The telescopes used were A 68 until 1869 Dec. 21 (except on 1863 Aug. 19), and E 80 from 1878. On 1863 Aug. 19, 1869 Dec. 26, and 1871 Mar. 16, C 39 was used. Dancer's eyepiece was used 1878 Mar. 21.

TABLE X.

Baxendell's Observations of R Persei.

Date.	J.D. 240+.	Corrected Comparisons.	Deducted Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deducted Mag.
1861				1862			
Oct. 9	1058	90+3, 96-4	92	Nov. 23	1468	90+4, 96-1	94
13	1062	90+2, 96-4	92	24	1469	90+3, 96-2	94
14	1063	90+2, 96-4	92	Dec. 26	1501	90+6, 101+1, 101	101
16	1065	90+2, 96-6	91			108-3	
Nov. 2	1082	96+1, 101-4	97	1863			
3	1083	96+2, 101-3	98	Jan. 23	1529	123+0, "f 12'3 mag. est."	123
11	1091	96+6, 101+0	102				
15	1095	101+7, 108+0	108	Aug. 19	1737	<110	
27	1107	108+9, 123-3	118	24	1742	"About 12½ mag., but ? right star"	
Dec. 3	1113	123-2	121	Oct. 9	1788	"Not sure if visible," <12¼	
21	1131	123+5	128				
25	1135	123+6	129	Nov. 2	1812	"Glimpsed occasion-ally," 134	
28	1138	Barely suspected,	132				
1862							
Oct. 18	1432	96+5, 101+0	101	28	1838	115+3, 123-7	117
29	1443	96+1, 101-6	96	29	1839	115+2, 123-6	117
Nov. 2	1447	90+5, 96-2	94	Dec. 5	1845	101+9, 115-2	112
10	1455	90+7, 96-2	96	17	1857	101-1	100
16	1461	90+5, 96-2	94	27	1867	96+2, 101-4	98

TABLE X.—*Baxendell's Observations of R Persei*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1864				1866			
Jan. 4	1875	96+1, 101-3	98	Oct. 1	2876	101+2, 104-1, ? 103 [108+0]	
28	1899	96+3, 101-2	99	6	2881	[108+0], 104-1, ? 104 101+3	
Feb. 5	1907	96+2, 101-1	99	9	2884	101+2, 104-3, ? 102 [108-1]	
Mar. 28	1959	Glimpsed < 127		14	2889	101-2	99
Aug. 23	2107	90+1, 96-3	92	19	2894	[108-2], 101-1, ? 100	
Sept. 14	2129	96+4, 101-6	98	28	2903	96+5, [108-3] ? 99 101-4	
24	2139	96+9, 101+0	103	Nov. 3	2909	96+2, [108-5] ? 96 101-8	
Dec. 30	2236	123+8	131	6	2912	96+0, [108-8] ? 96	
1865				11	2917	96+0	96
Jan. 20	2257	104+9, 123-9	114	19	2925	96+0	96
Feb. 8	2276	104+1	105	23	2929	96+1	97
14	2282	104+1	105	25	2931	96+1	97
Mar. 3	2299	90-1, 101-9	90	30	2936	96+0	96
9	2305	90-1	89	Dec. 8	2944	96+1	97
30	2326	90+5, 96-1, 101-3	96	16	2952	96+7, 101+0	102
Aug. 26	2475	96+3, 101-6	97	30	2966	115-2, 104+6, 113 123-7	
Sept. 27	2507	90+2, 96-3	92	1867			
Oct. 2	2512	90+3, 96-3	93	Feb. 2	3000	Glimpsed with diffi- culty	
19	2529	90+2, 96-3	92	20	3018	Not seen < 123+0	
Nov. 5	2546	96+1	97	28	3026	Glimpsed with diffi- culty	
26	2567	[108+1], 104+0, ? 104		Mar. 16	3042	Not seen < 123+0	
Dec. 22	2593	127		20	3046	„ „	
1866				26	3052	123+7, min. vis. 130 -6	
Feb. 12	2645	104+11, 123-7, 108+6	115	Aug. 1	3180	104+5, 115-1	112
17	2650	104+5, 123-9, 108+2	111	Oct. 23	3263	123+7, "f only glimpsed"	130
25	2658	108+1, 104+3, 96+8	107	1868			
Mar. 13	2674	96+0, 101-2	98	Feb. 3	3366	104-2	102
25	2686	90+6, 96-2	95	8	3371	101+2, 104-3	102
Apr. 13	2705	72+15, 90-3	87	11	3374	101+3, 104-1	104
Aug. 31	2845	Not seen < 11½					
Sept. 15	2860	123-6, 101+10	114 "Is d vari- able?"				
24	2869	104+6					

TABLE X.—*Baxendell's Observations of R Persei*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1868				1879			
Feb. 15	3378	104+0, 101+4	104	Mar. 1	7410	108+1, 123-9	112
19	3382	104+1	105	Aug. 23	7585	90+0, 96-2	92
Mar. 5	3397	104+13, 123-6	117	28	7590	96-1	95
July 25	3539	90+0	90	Sept. 13	7606	101+0, 108-4	102
Oct. 17	3623	Glimpsed (low) 13'0		25	7618	108+8, 123-8	116
Nov. 15	3652	Not seen < 12'5		Nov. 22	7676	Not seen < 123+0	
1869				Dec. 6	7690	„ „ < 123+4	
Feb. 5	3734	90+4, 96-2	94	1880			
6	3735	96-2	94	Mar. 17	7792	90+2, 96-2	93
Mar. 24	3781	90+4, 96-3	94	24	7799	90+4, 96+0	95
Oct. 26	3997	96+2, "hasty estimation"	98	Apr. 4	7810	96+7, 108-3	104
Dec. 21	4053	Not seen < 123+0		Aug. 2	7930	123-6	117
26	4058	„ < 11'0		11	7939	104-1	103
1871				14	7942	104+2	106
Mar. 16	4503	Not seen < 13'0		Sept. 3	7962	90+2, 96-2	93
1877				18	7977	72+11, 90-3	85
Dec. 2	6956	96-1	95	27	7986	72+9, 90-5	83
1878				Oct. 2	7991	72+10, 90-2	85
Mar. 21	7065	Invisible < 13'0		9	7998	90-2	88
Aug. 18	7215	Glimpsed < 123+0		29	8018	90+3, 96-3	93
25	7222	123+7	130	Nov. 20	8040	108+7, 123-10	114
Sept. 5	7233	Invisible < 123+5		Dec. 4	8054	123+3	126
23	7251	„ < 13'5		1881			
Oct. 27	7285	123+5, min. vis. -5	128	Feb. 28	8140	123-3	120
Nov. 8	7297	< 123+8, "but dif. to est.," "moonlight and haze"		Apr. 2	8173	96+3, 104+0, 101+1	102
19	7308	108+3, 123-13	110	Aug. 24	8317	< 13'5	
Dec. 5	7324	90+3, 96-2	94	31	8324	< 13'5	
8	7327	96-1	95	Sept. 14	8338	123+8	131
23	7342	90+0, 96-5	90	Oct. 1	8355	123-7	116
30	7349	90+3, 96-3	93	15	8369	104+3, 108+1	108
1879				20	8374	96+4, 108-4, 104+0	103
Jan. 25	7375	96-1	95	26	8380	96+5, 101+0, 104-3	101
Feb. 16	7397	101+7, 108-4	106	29	8383	96+4, 104-6	99
				Nov. 14	8399	90+1, 96-3	92
				17	8402	90-1	89

TABLE X.—*Baxendell's Observations of R Persei*—continued.

Date.	J.D. 2400+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 2400+.	Corrected Comparisons.	Deduced Mag.
1881				1884			
Nov. 20	8405	90 - 1	89	Nov. 17	9498	96 + 5	101
28	8413	90 + 0, 96 - 5	90	20	9501	96 + 3	99
Dec. 19	8434	90 + 2, ? 96 - 3	? 92	Dec. 4	9515	104 + 7, 117 - 6, 108 + 3	111
27	8442	90 + 3, 96 - 1	94	14	9525	123 - 6	117
1882				1885			
Sept. 9	8698	< 123 + 6.		Jan. 6	9548	123 + 9	132
1883				Feb. 19	9592	< 123 + 4	
Feb. 22	8864	96 + 0	96	Mar. 27	9628	123 - 7, 115 + 1	116
Mar. 3	8873	96 + 2	98	Apr. 19	9651	96 + 0	96
26	8896	123 - 6	117	Aug. 14	9768	Glimpsed doubtfully < 13'5	
Apr. 1	8902	123 + 1	124	Sept. 5	9790	Invisible < 13'5	
Aug. 25	9048	72 + 11, 90 - 3	85	8	9793	" "	
1884				Oct. 29	9844	96 + 2	98
Feb. 2	9209	96 + 7, 101 - 1	102	Nov. 8	9854	90 + 3, 96 - 3	93
10	9217	90 + 4, 96 - 3	94	15	9861	90 - 1	89
14	9221	96 - 3	93	Dec. 1	9877	90 - 6	84
24	9231	72 + 12, 90 - 3, 96 - 6	87	26	9902	90 + 2	92
Mar. 3	9239	90 - 2, 96 - 7	88	1886			
5	9241	90 - 5	85	Jan. 6	9913	90 + 5, 96 - 3	94
16	9252	90 + 2, 96 - 6	91	Feb. 1	9939	115 + 1, 123 - 7	116
21	9257	90 - 2	88	14	9952	123 + 4	127
23	9259	90 + 1, 96 - 5	91	Mar. 8	9974	Glimpsed, say 13'7 241 +	
31	9267	90 + 5, 96 - 4	94	Sept. 22	0172	Glimpsed 123 + 7	130
Apr. 12	9279	96 - 2, 101 - 3	96	Oct. 23	0203	Invisible < 123 + 4	
Aug. 14	9403	104 + 9, 123 - 11	112	Nov. 2	0212	Suspected 123 + 11 ? 134	
18	9407	104 + 6, 123 - 9	112	16	0227	123 + 9, 132 + 0	132
23	9412	104 + 3, 108 - 1	107	Dec. 4	0245	115 + 3, 123 - 7	117
Sept. 11	9431	96 + 2, 101 - 5	97	29	0270	101 + 0	101
17	9437	90 + 0, 96 - 5	90	1887			
20	9440	90 + 0	90	Jan. 12	0284	72 + 7, 90 - 6	82
26	9446	72 + 11, 90 - 5	84	29	0301	72 + 9, 90 - 8	82
29	9449	72 + 11, 90 - 5	84	Feb. 10	0313	72 + 9, 90 - 8	82
Oct. 1	9451	72 + 11, 90 - 7	83	18	0321	90 + 1	91
9	9459	72 + 11, 90 - 7	83	27	0330	90 + 6, 96 - 4	94
12	9462	90 - 5	85				
Nov. 9	9490	90 + 2, 96 - 3	92				
13	9494	96 + 2	98				

* The letter is *e*, but it seems almost certain that it should be *c*, and the observation has been amended accordingly.

These observations were treated in the usual manner, with an assumed period of 210 days. The means of different groups of periods are as follows, the unit being 0.1 magnitude :—

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
A (I. to VIII.) . . .	104 ₄	100 ₃	94 ₄	94 ₅	98 ₄	104 ₃	121 ₂	128 ₂₁	...	...	121 ₂	113 ₁
B (IX. to XV.) . . .	96 ₃	91 ₃	95 ₂	100 ₃	110 ₃	117 ₁	...	...	...	130 ₂	112 ₁	102 ₁
C (XXIX. to XXXVI.)	114 ₄	98 ₄	90 ₃	87 ₂	94 ₅	99 ₄	114 ₃	128 ₂	...	...	...	129 ₁
D (XXXVIII. to XLV.)	114 ₂	101 ₅	88 ₅	85 ₅	90 ₂₃	96 ₅	113 ₂₂	121 ₃	131 ₂	..	? 134 ₁	132 ₁
Mean with portions shifted.	101	93	89	93	98	110	122	130	...	? 134	128	113
No. of periods . . .	14	14	14	15	16	11	8	3	0	1	7	10

There is an evident retrograde movement of the maximum between A and B of rather more than one portion (say 22 days), and a forward movement of nearly two portions between B and C, while D is almost the same as C. The mean has therefore been taken by shifting the portions in B one step forward, and those in C and D one step back. This gives a good mean light curve, with a slight check in the fall about 20 days after maximum. The epoch of maximum, as read from the curve, is about 43 days after the zero point—the initial date of the period. One portion is unobserved, and another contains but one merely glimpsed observation, owing to the fact that Baxendell could not see the star when at its minimum, but the gap is easy to fill in with sufficient probability.

Corrections to epoch have been obtained in the usual manner in all cases where there are observations available for both rise and fall, and also for Period I., in which the rise is not observed, so that its result is very doubtful. General differences in brightness between different periods have been taken account of. Table XI. gives the comparisons of the observations with the mean curve, and Table XII. the corrections to epoch. The observations have been shifted in the manner already indicated, the first shift beginning with the later observations of Period VIII. The maximum point has been brought as nearly as possible into the middle of the portions used, at the cost, sometimes, of using fewer observations.

TABLE XI.

Comparison of the Observations with Mean Light Curve.

Period.	Initial Date 240 +.	Numbers and Mean Magnitude of Portions.											
		1. 101.	2. 93.	3. 89.	4. 93.	5. 98.	6. 110.	7. 122.	8. 130.	9. (134.)	10. 134.	11. 128.	12. 113.
I.	1000	...	...	...	-2	-3	-3	0	-2	...	...	...	...
III.	1420	+2	+2	+1	...	+1	...	+3	...	...	...	...	...
IV.	1630	...	...	...	...	...	...	...	...	...	...	...	+9
V.	1840	+8	+4	+7	+5.5	...	...	...	...	...	...	...	...
VI.	2050	...	...	...	+0.5	-1	-2	...	...	...	...	+4	+5
VII.	2260	+8	+10	+0.5	+1	...	...	...	...	...	...	...	...
VIII.	2470	-6	...	+3	0	0	-7	...	...	...	...	...	-4
IX.	2680 + 18	-2	+1	-2	...	...	...	...	...	...	...	...	0
X.	2890 „	+2	+6	+6	+4.5	+2	+5	...	...	...	...	0	...
XI.	3100 „	...	...	...	...	...	+1	...	...	...	...	+1	...
XII.	3310 „	...	...	...	...	+4	-1	0	...	...	...	...	...
XIII.	3520 „	...	...	0	...	...	...	...	...	...	...	...	...
XIV.	3730 „	...	0	...	-2	...	...	...	...	...	...	...	...
XV.	3940 „	...	...	...	...	+1	...	...	...	...	...	...	...
XIX.	6880 - 18	...	...	...	+3	...	...	...	...	...	...	...	...
XXX.	7090 „	...	...	...	...	...	...	+8	...	...	...	-3	-4
XXI.	7300 „	-8	-3	...	+3.5	+8	+4	...	...	...	...	...	...
XXII.	7510 „	...	...	...	+1	+5	+10	...	...	...	...	...	...
XXIII.	7720 „	...	...	...	+3	...	...	...	...	...	...	...	-7
XIV.	7930 „	-5	-7	-4	...	-3	+7	+8	...	...	...	...	-1
XXV.	8140 „	+4	...	...	...	...	...	...	...	...	...	+1	0
XVI.	8350 „	+1	0	-0.5	-3	-3	...	...	...	...	...	...	...
VIII.	8770 „	...	...	...	...	-3	...	-2	...	...	...	...	...
XIX.	8980 „	...	...	-4	...	...	...	...	...	...	...	...	...
XL.	9190 „	-6	-5	+1	+1	0	...	...	...	...	...	...	-4
XLI.	9400 „	-2	-7	-5	...	-1	0	-2	+2	...	...	...	...
XLII.	9610 „	+9	+2	...	...	...	...	...	...	...	...	...	...
XLIII.	9820 „	-4.5	-5	-6	-2	-3	+1	+5	...	...	...	...	...
LIV.	0030 „	...	...	...	...	...	...	...	+3	...	0	+2	0
XLV.	0240 „	+2	-12	-8	-6	-2	...	...	...	...	...	...	...
<i>p-f.</i>		+20	+12	0	-9	-17	-24	-20	-12	(-4)	(+6) +12	+21	+27

TABLE XII.

Corrections to Epoch.

Period.	Correction.	Period.	Correction.
I.	? + 7.4	XXXI.	+ 3.4
III.	+ 0.3	XXXIV.	+ 9.4
V.	+ 1.2	XXXVI.	+ 19.9
VIII.	- 5.2		or + 12.0
IX.	- 15.6	XL.	+ 11.5
X.	- 17.5	XLI.	+ 15.8
XI.	- 18.6	XLIII.	+ 13.4
XII.	- 19.6	XLV.	+ 17.4

The correction obtained in the usual way for Period XXXVI.—viz. + 19.9—is not in agreement with the observations, which suggest a maximum about 7 or 8 days earlier, with a possible error of several days. All the other epochs are compatible with the observations, though a difference of several days would sometimes be quite *possible*. Chandler's formula is :

$$2401039 + 210.3 E + 15 \sin (8^\circ E + 120^\circ),$$

but a revised form suits these epochs better, viz. :

$$2401039.7 + 210.0 E + 19 \sin (8^\circ E + 144^\circ).$$

These are both compared with the above epochs in Table XIII. The date obtained for the epoch of Period XLV. is probably too late. The correlation formulæ (*M.N.*, lxviii. p. 551) would give

$$21 \sin (6^\circ.4 E + \text{const.}).$$

TABLE XIII.

Period.	Epoch (Bax.)	Chandler. Bax.-Ch.	Epoch by New Formula.	B-n.f.	Discontinuity.	B-D.
I.	? 1050.4	1052.0 - 1.6	1050.9	- 0.5	1051.0	- 0.6
III.	1463.3	1470.0 - 6.7	1466.2	- 2.9	1465.8	- 2.5
V.	1884.2	1887.2 - 3.0	1881.0	+ 2.2	1880.6	+ 3.6
VIII.	2507.8	2512.1 - 4.3	2503.2	+ 4.6	2502.8	+ 5.0
IX.	2707.4	2720.4 - 13.0	2710.8	- 3.4	2710.2	- 2.8
X.	2915.5	2928.6 - 13.1	2918.5	- 3.0	2917.6	- 2.1
XI.	3124.4	3136.9 - 12.5	3125.5	- 1.1	3125.0	- 0.6
XII.	3333.4	3345.3 - 11.9	3332.6	+ 0.8	3332.4	+ 1.0
XXXI.	7346.4	7348.0 - 1.6	7347.4	- 1.0	7345.2	+ 1.2
XXXIV.	7983.4	7985.0 - 1.6	7983.8	- 0.4	7978.8	+ 4.6
XXXVI.	$\left. \begin{array}{l} 8412.9 \\ \text{or} \\ 8405.0 \end{array} \right\}$	$\left. \begin{array}{l} 8409.1 \\ \left\{ \begin{array}{l} + 3.8 \\ \text{or} \\ - 4.1 \end{array} \right\} \end{array} \right\}$	$\left. \begin{array}{l} 8406.8 \\ \left\{ \begin{array}{l} + 6.1 \\ \text{or} \\ - 1.8 \end{array} \right\} \end{array} \right\}$	$\left. \begin{array}{l} 8401.2 \\ \left\{ \begin{array}{l} + 11.7 \\ \text{or} \\ + 3.8 \end{array} \right\} \end{array} \right\}$		
XL.	9244.5	9267.0 - 22.5	9248.6	- 4.1	9246.0	- 1.5
XLI.	9458.8	9477.8 - 19.0	9458.1	+ 0.7	9457.2	+ 1.6
XLIII.	9876.4	9898.5 - 22.1	9876.2	+ 0.2	9877.0	- 0.6
XLV.	0300.4	0319.3 - 18.9	0292.9	+ 7.5	0296.8	+ 3.9
	or 0288.0			or - 4.9		or - 8.8

To test the formula more thoroughly we must have recourse to other observations. In *H.A.*, lv. p. 128, are collected the dates of a number of maxima from 1861 to 1900, including Baxendell's. They are in good accord with the proposed formula for the early years, but not at the end. The term $+19 \sin (8^\circ E + 144^\circ)$ vanishes at $E = 4.5, 27.0, 49.5, 72.0$, etc. etc. Between the first two, *i.e.* from J.D. 2401950 to 2406675, the term is negative, and the other observations agree, as will be seen below. From 2406675 to 2411400 it is positive; but after 2411400 it should again be negative, up to 2416125, whereas the 9 maxima quoted in *H.A.* are all consistently positive. Indeed, the period from about 2409400 seems to be steadily 210 days to a close approximation.

The suggestion is therefore put forward that the results can best be represented by three straight lines; *i.e.* there have been two changes of period.

E	Max.	Period.	Correction to Uniform Period.
1	2401051.0	207.4	+ 8.0
12	2403332.4	211.2	- 20.6
41	2409457.2	209.9	+ 14.2
85	2418692.8		+ 9.8

Baxendell's observations are compared with this "discontinuity" hypothesis in the last two columns of Table XIII. The numerical sum of the residuals is about the same as for the suggested new formula. But it is in comparison with observations later than Baxendell's that the discontinuity formula shows to advantage. The maxima tabulated in *H.A.*, lv. p. 128, show the following deviations from it:—

E.	H.A.	E.	H.A.	E.	H.A.	E.	H.A.	E.	H.A.
	d		d		d		d		d
0	+ 14	9	+ 5	21	- 5	40	- 1	61	- 11
2	0	13	+ 3	23	- 11	42	+ 5	63	- 1
4	- 2	14	+ 5	32	+ 6	44	- 10	67	- 5
5	+ 16	16	+ 1	33	+ 9	52	- 1	75	0
6	+ 11	18	- 10	35	+ 14	53	+ 5	...	...
7	+ 10	19	+ 1	37	+ 7	56	- 11	...	...
8	+ 2	20	+ 2	39	- 5	58	- 2	...	...

Analysis of Baxendell's mean light curve gives the expression $23.12 \cos (\theta - 245^\circ) + 1.014 \cos (2\theta - 205^\circ) + 1.014 \cos (3\theta - 171^\circ)$, which becomes, when divided by the range and multiplied by 6,

$$3.02 \cos (\theta - 245^\circ) + 0.13 \cos (2\theta - 205^\circ) + 0.13 \cos (3\theta - 171^\circ).$$

Each of the groups has also been analysed, and the results are given below:—

Mean Curve: $3.02 \cos (\theta - 245^\circ) + 0.13 \cos (2\theta - 205^\circ) + 0.13 \cos (3\theta - 171^\circ)$

Group A: $2.98 \cos (\theta - 254^\circ) + 0.24 \cos (2\theta - 142^\circ) + 0.26 \cos (3\theta - 126^\circ)$

„ B: $2.89 \cos (\theta - 218^\circ) + 0.38 \cos (2\theta - 138^\circ) + 0.19 \cos (3\theta - 45^\circ)$

„ C: $3.03 \cos (\theta - 270^\circ) + 0.16 \cos (2\theta - 288^\circ) + 0.21 \cos (3\theta - 281^\circ)$

„ D: $3.04 \cos (\theta - 276^\circ) + 0.28 \cos (2\theta - 314^\circ) + 0.12 \cos (3\theta - 261^\circ)$

As a greater amount of the curve has had to be filled in for these groups, the results are not very trustworthy; but the changes in phase of the principal term agree well with the corrections to epoch, the difference in A from the mean curve working out at +5 days, that in B at -16 days, that in C at +14½ days, and that in D at +18 days. Of course all the periods are here used, not only those which appear in the corrections.

[*Note added April 6.*—There are a number of Harvard observations of these two stars given in *H.A.*, lxiii. These have been reduced since the above was set in type, and as they throw new light on the possibilities, a brief account is given here.

As regards T Herculis, the Harvard observations confirm those of the B.A.A. in suggesting that the period has recently been sensibly constant: certainly there have been no changes of 16, or even of 8 days, such as are suggested by Baxendell's observations. While the mean results agree perfectly with those of the B.A.A., there is no confirmation of the apparent exceptional epochs of CVI. and CIX.; and a glance at the B.A.A. curves shows that the disregard of a very few observations would dispose of these exceptions, while that of XCII. is actually contradicted by the appearance of the curve.

The following table summarises the results:—

Period.	Baxendell Epoch.	B.A. Calculated Correction.	Harvard Calculated Correction.	Chandler.	$\frac{1}{2}(\text{B.A.} + \text{H.})$ - Chandler.
	241 +				
XCII.	5283	+ 11·8	+ 6·0	5288·4	+ 3·5
XCIV.	5613	+ 2·6	+ 7·0	5617·7	+ 0·1
XCV.	5778	...	+ 12·0	5782·2	(+ 7·8)
XCVI.	5943	+ 0·7	- 2·0	5946·7	- 4·5
XCVIII.	6273	...	- 2·0	6275·4	(- 4·4)
CI.	6768	+ 3·0	+ 1·5	6768·2	+ 2·0
CII.	6933	...	+ 3·0	6932·4	(+ 3·6)
CIII.	7098	+ 5·8	+ 4·0	7096·6	+ 6·3
CIV.	7263	+ 2·5	+ 5·8	7260·7	+ 6·4
CV.	7428	+ 3·5	+ 8·5	7424·9	+ 9·1
CVI.	7593	+ 12·0	+ 6·8	7589·0	+ 13·4
CVII.	7758	+ 5·4	+ 6·5	7753·1	+ 10·8
CVIII.	7923	+ 6·1	+ 4·5	7917·3	+ 11·0
CIX.	8088	- 2·5	+ 3·3	8081·4	+ 7·0
CX.	8253	+ 6·1	+ 2·4	8245·6	+ 11·7
CXI.	8418	+ 7·6	+ 3·5	8409·8	+ 13·8
CXII.	8583	+ 1·3	+ 1·9	8574·0	+ 10·6
CXIII.	8748	...	+ 7·1	8738·3	(+ 16·8)
CXIV.	8913	...	+ 3·0	8902·6	(+ 13·4)
Mean XCII. to CXIV. }	...	+ 4·7	+ 4·4	...	+ 7·3
Mean XCII. to CIII. }	...	+ 4·8	+ 3·7	...	+ 1·8
Mean CIV. to CXII. }	...	+ 4·7	+ 4·8	...	+ 11·3

The Harvard epochs as far as CIII. are those collected in *H.A.*, lv., from various sources, and estimated by the observers; from CIV. they are calculated from Harvard observations in *H.A.*, lxiii., by the methods adopted in this work.

In face of the apparent uniformity of period during these 22 periods, the Baxendell results have been carefully examined again, to ascertain whether the large corrections of periods XLVII. to LVIII. could possibly be spurious; but it will be seen on reference to the observations that unless they are held to be erroneous, the corrections must stand. It is of course possible that Baxendell changed his method of observation in some essential, in which case the changes are to be considered human and not stellar; but they are too large and definite to be treated as "accidental."

As regards R Persei, the following epochs of maximum have been calculated (by the process used in these papers) from the Harvard observations in *H.A.*, lxiii. They are compared with the "Discontinuity" hypothesis:—

E.	Harvard.	Harv. - Dis.
	d	d
77	7223'4	-0'1
79	7641'7	-1'6
80	7854'9	+1'7
81	8063'0	-0'1
82	8270'3	-2'7
84	8692'2	-0'6

A New Method of Discovering Periodicities.

By J. I. Craig, M.A., F.R.S.E.

(Communicated by Professor H. H. Turner.)

[The proof and MS. of this paper were sent to the author more than a year ago, but have not yet been returned. It seems undesirable to await them longer. A few obvious corrections have been made, and Professor H. C. Plummer has kindly added a note on § 5 at the end.—H. H. Turner.]

1. Various methods for detecting periodicity have been proposed from the time of Lagrange, and the best known is that given by Professor Schuster and improved recently by Professor Turner—the method of the periodogram. Another method has lately been published by Professor Dale, resembling in some respects the method here sketched, which was originally outlined at a meeting of the Cairo Scientific Society in February 1914. The full mathematical theory was elaborated and sent to Professor

Turner for publication before Professor Dale's paper came into the writer's hands.

2. In common with others, the writer considers that in meteorology we have to do with the state of steady motion summed up in climate, and the deviations from it that give weather. But deviations from a state of steady motion are generally oscillatory, and when they are their investigation depends on differential equations of the type

$$\frac{d^2x}{dt^2} + n^2x = 0.$$

This is a linear equation connecting the displacement from a mean position (deviation) with its acceleration. It is a linear regression equation of a kind rendered familiar in recent statistical mathematics.

In practice, however, our data are usually discrete, not continuous. Differentiation is difficult, and is very sensitive to errors of observation, and differential coefficients must be replaced by finite differences.

3. Periodicity of a function y_k is typified by an equation of the form

$$y_k = A \cos ka + B \sin ka,$$

where $2\pi/a$ is the period. The finite difference equation corresponding to this is

$$\Delta^2 y_k + 4 \sin^2 \frac{a}{2} y_{k+1} = 0.$$

Again, a linear regression equation between y_{k+1} and the second difference of y_k . The constant $\lambda = 4 \sin^2 \frac{a}{2}$ may be found by the method of correlation or by least squares. If λ (as may happen) is not positive and less than 4, other interesting cases arise that will not be dealt with here, but are treated in the full paper.

For calculation (and theory also) it is better to replace this equation by its equivalent

$$Ky_k - \cos a y_k = 0,$$

where K signifies the operation of taking the mean of the two values of y , one on each side of y_k . Nothing could be simpler than the procedure thus indicated, and it would be final but for the "effect" of errors of the observed quantities.

4. In general, y_k is not exactly representable by $A \cos ka + B \sin ka$, even where there is only one period, but

$$y_k = A \cos ka + B \sin ka + v_k,$$

where v_k is the residual. If now we operate by $K - \cos a$ on this equation we obtain

$$Ky_k - \cos a y_k = Kv_k - \cos a v_k.$$

Squaring this equation and summing for all available values (*i.e.* $n-2$ of the given n), we get

$$\begin{aligned}\Sigma(Ky_k)^2 - 2 \cos \alpha \Sigma(y_k Ky_k) + \cos^2 \alpha \Sigma y_k^2 \\ = \Sigma(Kv_k)^2 - 2 \cos \alpha \Sigma(v_k Kv_k) + \cos^2 \alpha \Sigma(v_k)^2.\end{aligned}$$

The left-hand side may be written

$$(n-2)(\sigma_1^2 - 2\sigma_0\sigma_1r_{01}\cos\alpha + \sigma_0^2\cos^2\alpha),$$

where σ_0 and σ_1 are the standard deviations of y_k and Ky_k , here taken to be deviations, as is allowable, and r_{01} is the coefficient of correlation between y_k and Ky_k . As yet we know nothing of the residuals, but we may assume that, on a large series of observations, the following relations are verified, with negligible errors:

$$\Sigma v_{k+1}^2 = \Sigma v_k^2 = \Sigma v_{k-1}^2 = (n-2)\sigma_v^2$$

$$\Sigma(Kv_k^2) = \frac{1}{4}\Sigma(v_{k+1} + v_{k-1})^2 = \frac{1}{2}(n-2)\sigma_v^2(1 + \rho''),$$

where ρ'' is the coefficient of correlation between v_{k+1} and v_{k-1} .

Similarly,

$$\Sigma v_k Kv_k = (n-2)\sigma_v\sigma_{kv}\rho',$$

where ρ' is the coefficient of correlation between v_k and Kv_k . The right-hand side then becomes

$$(n-2) \left\{ \frac{1}{2}(1 + \rho'') - 2 \left(\frac{1 + \rho''}{2} \right)^{\frac{1}{2}} \rho' \cos \alpha + \cos^2 \alpha \right\} \sigma_v^2.$$

Hence, noticing that $\sigma_v^2 = \frac{1}{n} \Sigma v^2$, *i.e.* one n -th of the function which is made a minimum in the method of least squares, and writing U for it, we have

$$\sigma_1^2 - 2\sigma_0\sigma_1r_{01}\cos\alpha + \sigma_0^2\cos^2\alpha = U \left\{ (1 + \rho'')/2 - 2\rho' \cos\alpha \left(\frac{1 + \rho''}{2} \right)^{\frac{1}{2}} + \cos^2\alpha \right\},$$

which is the fundamental equation in what follows.

The only assumptions made so far are more and more closely justified as the number of observations increases, and when this is infinite, the errors involved in the assumptions vanish.

5. We come now to another sort of assumption analogous to those made in the method of solution of equations by successive approximation.

If there is one and only one period, as we are here supposing, there can be no correlation between v_k and Kv_k ; for this would lead to another period, since it reproduces the relationship on which the whole argument is based. We may therefore, as a first approximation, put ρ' zero. If, in the event, it should turn out different from zero by an amount that is significant, *vis à vis* the errors of observation, the meaning is that we must carry the process further and look for another period. We have thus a criterion of the (practical) finality of our process.

The second assumption refers to ρ'' . In the absence of all

information regarding the residuals, we may assume that the coefficient of correlation between alternate pairs is approximately equal to the coefficient for alternate pairs of the original observations, r'' say. But

$$\begin{aligned}\sigma_1^2 &= \frac{1}{n-2} \sum (Ky_k)^2 = \frac{1}{4(n-2)} \sum (y_{k+1} + y_{k-1})^2 \\ &= \frac{1}{2}(\sigma_0^2 + \sigma_0^2 r'').\end{aligned}$$

Hence

$$\frac{1}{2}(1 + r'') = \frac{1}{2}(1 + r'') = \sigma_1^2 / \sigma_0^2$$

as a working first approximation. The fundamental equation is now simplified to

$$\sigma_1^2 - 2\sigma_0\sigma_1r_{01}\cos a + \sigma_0^2\cos^2 a = (\sigma_1^2/\sigma_0^2 + \cos^2 a)U,$$

and U is to be made a minimum.

Known methods for this * give the following condition, since

$$\delta U / \delta \cos a = 0, -\sigma_0\sigma_1r_{01} + \sigma_0^2\cos a = U\cos a,$$

and on subtracting this equation, multiplied throughout by $\cos a$, from the fundamental equation, we obtain

$$\sigma_1^2 - \sigma_0\sigma_1r_{01}\cos a = (\sigma_1^2/\sigma_0^2)U.$$

Elimination of $\cos a$ gives the equation for U

$$\begin{vmatrix} \sigma_0^2 - U & \sigma_0\sigma_1r_{01} \\ \sigma_0\sigma_1r_{01} & \sigma_1^2 - U\sigma_1^2/\sigma_0^2 \end{vmatrix} = 0$$

or

$$(\sigma_0^2 - U)^2 - \sigma_0^4 r_{01}^2 = 0,$$

whence

$$\cos a = \sigma_0\sigma_1r_{01}/(\sigma_0^2 - U) = \pm \sigma_1/\sigma_0,$$

that sign being selected which gives $\cos a$ the same sign as r_{01} , since $\sigma_0^2 - U$ is necessarily positive.

The first approximation to a is therefore got from the remarkable formula just quoted. The period is then given by $p = 2\pi/a$.

6. It remains to find the first approximations to the amplitude and phase.

Hitherto we have supposed y_k to be a deviation from its mean. It is not difficult to show that the above argument holds when y_k is the actual observed quantity and the equation is

$$y_k = A \cos ka + B \sin ka + \bar{y},$$

where $\bar{y}$ is a mean, but not necessarily the average of the y 's, unless the series covers exactly an exact number of periods, p . The normal equations in the method of least squares are

$$\begin{aligned}A \sum \cos^2 ka + B \sum \cos ka \sin ka + \bar{y} \sum \cos ka &= \sum y_k \cos ka \\ A \sum \cos ka \sin ka + B \sum \sin^2 ka + \bar{y} \sum \sin ka &= \sum y_k \sin ka \\ A \sum \cos ka + B \sum \sin ka + n\bar{y} &= \sum y_k.\end{aligned}$$

* This method is selected because it admits of easy generalisation.

If we write $F(a) = \sin na / \sin a$, we have by actual summation,

$$A\left(\frac{1}{2}n + \frac{1}{2}F(a)\cos(n+1)a\right) + B\left(\frac{1}{2}F(a)\sin(n+1)a\right) + \bar{y}F\left(\frac{a}{2}\right)\cos\frac{n+1}{2}a = \Sigma y_k \cos ka$$

$$A\left(\frac{1}{2}F(a)\sin(n+1)a\right) + B\left(\frac{1}{2}n - \frac{1}{2}F(a)\cos(n+1)a\right) + \bar{y}F\left(\frac{a}{2}\right)\sin\frac{n+1}{2}a = \Sigma y_k \sin ka$$

$$A\left(F\left(\frac{a}{2}\right)\cos\frac{n+1}{2}a\right) + B\left(F\left(\frac{a}{2}\right)\sin\frac{n+1}{2}a\right) + n\bar{y} = \Sigma y_k.$$

When the number of observations is large, the terms in $F(a)$ may be neglected, and contracted methods are available for computing the absolute terms on the right hand. For example,

$$\Sigma y_k \cos ka = nr_{\gamma}\sigma_0\sigma_{\gamma}$$

where r_{γ} is the coefficient of correlation between the original observations and the series $\Sigma \cos ka$, and σ_{γ} is the standard deviation of this series. Or approximate harmonic analysis may be used.

A , B , and $\bar{y}$ having been found,

$$R^2 = A^2 + B^2, \text{ and } \tan \beta = B/A, \text{ and } y_k = R \cos(ka - \beta) + \bar{y},$$

the approximate solution of the problem.

7. It remains to be seen how far the second series of assumptions is justified. If it is, we have the best solution of the problem; if not, a correction becomes necessary.

The residuals $v_k = y_k - R \cos(ka - \beta) - \bar{y}$ may be calculated, and the coefficients of correlation ρ' and ρ'' obtained. If ρ' differs from zero by an insignificant amount, our assumption of a single period is justified; but if not, it is necessary to search for a second period, in which case it is better to begin again and employ the theory for two periods. If, however, we decide to stop at one period, it appears to be more legitimate to put zero for ρ' than the value that arises from calculation.

We may now substitute for ρ'' the better value got from calculation. The equation for U then becomes

$$\begin{vmatrix} \sigma_0^2 - U & \sigma_0\sigma_1r_{01} \\ \sigma_0\sigma_1r_{01} & \sigma_1^2 - \frac{1}{2}(1 + \rho'')U \end{vmatrix} = 0$$

and for $\cos a$, $\cos a(\sigma_0^2 - U) = \sigma_0\sigma_1r_{01}$, as before, but with a slightly different value of U .

It is possible to find corrections δU and δa to U and a , but in a case like the present it is perhaps simpler to solve the equations again.

If the change in a is considerable, it may sometimes be necessary to proceed to a third approximation.

The rigorous method of procedure when there are several periods is given in the fuller paper mentioned above.

8. The method is here applied to find the best single period for sun-spot numbers. These have been given and discussed by

Professor A. Schuster, F.R.S., in his paper (*Phil. Trans.*, A, 206, p. 96) and by Professor H. H. Turner, F.R.S. (*M.N.*, *R.A.S.*, lxxiv., 2, p. 82). Following the former, the writer has divided the series from 1750 to 1899 into two, viz. 1750-1824 and 1825-1899, but the calculations had advanced too far before it was noticed that Professor Schuster's two partial series differ slightly from these. The effect of this difference cannot be great.

The results are :

Series.	α .	p .
1750-1899	31'200	11'538 years
1750-1824	31'217	11'532 ,,
1824-1900	31'778	11'329 ,,

The chief period for sun-spots is usually considered to be 11'125 years, but Professor Turner (*M.N.*, *R.A.S.*, lxxiii., No. 9, pp. 719 and 726) gives reasons for supposing that the real period is in the neighbourhood of 11'5 years, a value which is confirmed by the above results.

In another artificial example, in which the series is constructed on the formula $100 \sin 40k + v_k$, where v is an arbitrarily chosen number, this method gives $\alpha = 40^\circ 58$, while a more rigorous examination gives $40^\circ 42$, on the second approximation.

9. The method therefore works in practice. It is simple and straightforward, and the labour is so much less than that required for the method of the periodogram that there is now no excuse if claims for periodicity are put forward without analytical support.

Further, the method affords a criterion when practical finality has been attained; but this demands the consideration of the precision of the constants, which is left over for the present.

In conclusion, it may be pointed out that the theory is adaptable to expression of a series in terms of any functions defined by linear finite difference or functional equations.

[*Note by Professor H. C. Plummer.*—The argument of § 5 seems a little obscure. In particular the second assumption seems distinctly artificial as a starting point in the process of successive approximation. Something of its nature can be seen from the following consideration of its consequences. Since

$$\frac{1}{2}(1 + \rho'') = \frac{1}{2}(1 + r'') = \sigma_1^2 / \sigma_0^2$$

and

$$\cos \alpha = \pm \sigma_1 / \sigma_0,$$

it follows that

$$\cos 2\alpha = r'' = \rho''.$$

But ρ'' is the coefficient of correlation between v_{k+1} and v_{k-1} , i.e. between series of numbers which are essentially independent of the period. The assumption as to ρ'' is therefore clearly unsound, and can only be supported on the ground that it is necessary to make some assumption and to hope that the subsequent

process of approximation will converge to the right period. Apart from some uncertainty whether this hope will be realised, the process appears more laborious than the author implies.

Perhaps an alternative way of looking at the question may be clearer. In the first place, let the residuals be neglected altogether. Then

$$\sigma_1^2 - 2\sigma_0\sigma_1r_{01}\cos\alpha + \sigma_0^2\cos^2\alpha = 0.$$

But in this case

$$Ky_k - \cos\alpha y_k = 0,$$

and therefore $r_{01} = 1$. Hence immediately

$$\cos\alpha = \pm\sigma_1/\sigma_0,$$

the same first approximation as that arrived at in § 5. The rest of the process follows as in the paper. This seems a simpler point of view, though of course it makes no practical difference.]

Causes contributory to the Annual Variation of Latitude.

By Harold Jeffreys, B.A., M.Sc. (Plate 8.)

The earliest investigations of Chandler into the motion of the terrestrial pole relative to the surface of the earth showed that it was composed mainly of two parts. One is a circular motion, of period 430 days. This has always been considered to be identical with the free vibration called the polhode motion, or Eulerian nutation. The other is an annual movement in an ellipse. On account of the important information about the rigidity of the earth given by the Eulerian period, the explanation of the annual terms seems to have been very much neglected. It is generally attributed to meteorological causes, but, so far as I have found, there has been no attempt to investigate whether these causes are quantitatively capable of accounting for the observed effect, save a partial one by Herr R. Spitaler in 1897.

It is easy to see how variations in the position of the axis of rotation can arise. Consider a heavy particle attached to the earth in the Greenwich meridian. On account of rotation, a tension between this particle and the earth is necessary to keep it moving in a circle about the axis of rotation. This tension is perpendicular to the axis of rotation, and if the particle is not in the equator, the tension will have a moment about an axis in the equator and perpendicular to the Greenwich meridian. Thus a couple must act on the earth when any surface deformation takes place. Further, as this couple is found to depend only on the amount of this deformation, its effect in altering the moments of momentum must be proportional to the time for which it acts. Hence we should

expect *a priori* that even if a semi-annual or daily displacement were as great as an annual displacement, yet, owing to the longer time the effect of the latter has to accumulate, the annual part would have a much greater effect on the angular velocities, and hence on the position of the instantaneous axis. Owing to the fact that the free period is fairly near to a year, the annual part is still further magnified by resonance. This may contain the explanation of the smallness of the annual parts of the couples found by Sir Joseph Larmor and Colonel Hills.* Irregular parts in the couples, roughly periodic in six months, say, would be diminished about sixfold in comparison with the annual parts when the motion of the instantaneous axis is calculated from them.

I. *The Effect of Meteorological Changes on the Earth's Rotation.*

Let a system of axes at the centre of the earth be defined as follows:—

The axis of x is in the plane of the equator, and intersects the meridian of Greenwich.

The axis of y is in the plane of the equator, and is 90° east of Ox .

The axis of z is the axis pointing to the mean position of the north pole.

Let the angular velocities of these axes, relative to axes fixed in space and instantaneously occupying the same positions, be $(\theta_1, \theta_2, \theta_3)$.

Let A', B', C' be what the moments of inertia of the earth would become if rotation were removed. Let the products of inertia be F, G, H . Also let I be the additional moment of inertia about the axis of rotation that is caused by the elastic strain of rotation. This strain takes the form of an equatorial protuberance, symmetrical about the instantaneous axis. If n is the resultant angular velocity of the earth, the direction cosines of the instantaneous axis are $(\theta_1/n, \theta_2/n, \theta_3/n)$. The angular momentum is then composed of three parts.

(1) The part due to the motion as if the earth were a rigid body, rigidly attached to the axes of reference, and with moments and products of inertia $(A'B'C'FGH)$, has components

$$A'\theta_1 - H\theta_2 - G\theta_3, \quad B'\theta_2 - F\theta_3 - H\theta_1, \quad C'\theta_3 - G\theta_1 - F\theta_2.$$

(2) The part due to rotational strain is of magnitude In , about the instantaneous axis. Its components are therefore $I\theta_1, I\theta_2, I\theta_3$.

(3) There is a part due to motion relative to the axes. In cases of simple deformation due to vertical movement the velocities involved in these displacements would necessarily have no moment about the centre of the earth. If matter was collecting at a point,

* *Monthly Notices*, vol. lxvii. 1906-7, p. 24; and vol. lxxv. 1914-15, p. 518.

the velocities of approach to the point being symmetrical about it, the momenta would likewise be zero. In all ordinary displacements it seems that this part is either zero or very small in comparison with $A'\theta_1$.* If it should subsequently be found that there are large periodic motions relative to the earth as a whole that do produce a considerable angular momentum, then these will have to be included in a more detailed treatment. They are not considered in the present paper.

If, then, the total angular momenta are (h_1, h_2, h_3) , we have

$$\left. \begin{aligned} h_1 &= (A' + I)\theta_1 - H\theta_2 - G\theta_3 \\ h_2 &= (B' + I)\theta_2 - F\theta_3 - H\theta_1 \\ h_3 &= (C' + I)\theta_3 - G\theta_1 - F\theta_2 \end{aligned} \right\} \quad . \quad . \quad . \quad (i.)$$

The equations of motion are therefore :

$$\left. \begin{aligned} \frac{d}{dt} \{ (A' + I)\theta_1 - H\theta_2 - G\theta_3 \} - \{ (B' + I)\theta_2 - F\theta_3 - H\theta_1 \} \theta_3 \\ \quad + \{ (C' + I)\theta_3 - G\theta_1 - F\theta_2 \} \theta_2 = 0 \\ \frac{d}{dt} \{ (B' + I)\theta_2 - F\theta_3 - H\theta_1 \} - \{ (C' + I)\theta_3 - G\theta_1 - F\theta_2 \} \theta_1 \\ \quad + \{ (A' + I)\theta_1 - H\theta_2 - G\theta_3 \} \theta_3 = 0 \\ \frac{d}{dt} \{ (C' + I)\theta_3 - G\theta_1 - F\theta_2 \} - \{ (A' + I)\theta_1 - H\theta_2 - G\theta_3 \} \theta_2 \\ \quad + \{ (B' + I)\theta_2 - F\theta_3 - H\theta_1 \} \theta_1 = 0 \end{aligned} \right\} \quad (ii.)$$

In the undisturbed state the earth may be regarded as dynamically symmetrical about the polar axis. Let the undisturbed values of A' , B' , C' be A_0 , A_0 , C_0 .

$$\text{Put} \quad A' = A_0 + A_1, \quad B' = A_0 + B_1, \quad C' = C_0 + C_1 \quad . \quad (iii.)$$

Then A_1 , B_1 , C_1 , F , G , H , θ_1 , θ_2 are all small quantities of the first order.

If we neglect their squares and products, the third equation of motion reduces to

$$\frac{d}{dt}(C\theta_3) = 0,$$

and therefore $C\theta_3$ is a constant, equal to $(C_0 + I)n$, the angular momentum due to rotation.

The first two equations become

$$\begin{aligned} (A_0 + I)\theta_1' - (C_0 + I)n \frac{d}{dt} \left(\frac{G}{C} \right) - (A_0 - C_0)n\theta_2 + Fn^2 &= 0 \\ (A_0 + I)\theta_2' - (C_0 + I)n \frac{d}{dt} \left(\frac{F}{C} \right) - (C_0 - A_0)n\theta_1 - Gn^2 &= 0 \end{aligned}$$

where accents denote differentiation with regard to the time.

* This point has been considered in greater detail by Sir G. H. Darwin, *Scientific Papers*, vol. iii. p. 40.

$$\text{Then } \left. \begin{aligned} \theta_1' + \frac{C_0 - A_0}{A_0 + I} n \theta_2 &= \frac{C_0 + I}{A_0 + I} n \frac{d}{dt} \left(\frac{G}{C} \right) - \frac{F n^2}{A_0 + I} \\ \theta_2' - \frac{C_0 - A_0}{A_0 + I} n \theta_1 &= \frac{C_0 + I}{A_0 + I} n \frac{d}{dt} \left(\frac{F}{C} \right) + \frac{G n^2}{A_0 + I} \end{aligned} \right\} \quad (\text{iv.})$$

These are the equations governing the variation of latitude. The period of the complementary functions, which cause the Eulerian nutation, is $2\pi(A_0 + I)/(C_0 - A_0)n$, and we must therefore attribute to $(C_0 - A_0)/(A_0 + I)$ the value $1/\tau$, where τ is the number of days in the Eulerian period. The terms on the right involving d/dt are smaller than the others in the ratio $\frac{1}{365}$ for the case of an annual motion. I/A_0 may also be omitted.

Finally, then, the equations of motion to be treated are

$$\left. \begin{aligned} \theta_1' + \frac{n\theta_2}{\tau} &= -\frac{F}{A_0} n^2 \\ \theta_2' - \frac{n\theta_1}{\tau} &= \frac{G}{A_0} n^2 \end{aligned} \right\} \quad (\text{v.})$$

The direction cosines (l, m, n) of the earth's axis of rotation are proportional to the angular velocities about the axes of reference. Their values are therefore $\left(\frac{\theta_1}{n}, \frac{\theta_2}{n}, 1 \right)$.

The Ellipsoid of Inertia.

If (λ, μ, ν) be the direction cosines of any diameter whatever of the ellipsoid after the centrifugal force has been removed, so that there is no rotational strain, the moment of inertia about it is

$$I = A'\lambda^2 + B'\mu^2 + C'\nu^2 - 2F\mu\nu - 2G\nu\lambda - 2H\lambda\mu \quad (\text{vi.})$$

This has a stationary value when

$$\frac{1}{\lambda} \frac{\partial I}{\partial \lambda} = \frac{1}{\mu} \frac{\partial I}{\partial \mu} = \frac{1}{\nu} \frac{\partial I}{\partial \nu} = \sigma \text{ say.}$$

These give

$$\left. \begin{aligned} (A' - \sigma)\lambda - H\mu - G\nu &= 0 \\ -H\lambda + (B' - \sigma)\mu - F\nu &= 0 \\ -G\lambda - F\mu + (C' - \sigma)\nu &= 0 \end{aligned} \right\} \quad (\text{vii.})$$

There is evidently an axis of maximum moment of inertia somewhere near the undisturbed one, namely, the z axis. Hence we can put λ and μ small, and $\nu = 1$.

Then, neglecting second-order terms,

$$\begin{aligned} (A_0 - \sigma)\lambda - G &= 0 \\ (A_0 - \sigma)\mu - F &= 0 \\ C_0 - \sigma &= 0. \end{aligned}$$

$$\text{Hence } (C_0 - A_0)\lambda = -G, \quad (C_0 - A_0)\mu = -F.$$

Thus the direction of the axis of the ellipsoid of inertia is known.

For the purposes of this investigation we shall find it convenient to *define* as the "pole of inertia" the end of the axis whose direction is obtained by putting $C_0 - A_0 = C_0/\tau$ in the above; so that the direction cosines of the axis of inertia are by definition

$$\lambda = -\frac{G\tau}{C_0}; \mu = -\frac{F\tau}{C_0}; \nu = 1. \quad (\text{viii.})$$

This axis is only used as an intermediary in the course of the work.

The observations of the earth's rotation give the direction of the axis of rotation directly. Let its direction cosines be ($l, m, 1$).

Then

$$\left. \begin{aligned} \lambda &= -\frac{m'\tau}{n} + l \\ \mu &= +\frac{l'\tau}{n} + m \end{aligned} \right\} \quad (\text{ix.})$$

These serve to find λ and μ when l and m are known. Our object is to compare the values thus obtained with those derived from the theoretical values of F and G . As l, m, λ, μ are all small quantities they can be replaced by the angles whose sines are l, m, λ, μ respectively without introducing error.

According to Kimura* the annual portion of the variation of latitude was given by

$$\left. \begin{aligned} x &= 0''.011 \sin \odot - 0''.096 \cos \odot \\ y &= 0''.051 \sin \odot + 0''.004 \cos \odot \end{aligned} \right\} \text{from } 1893.8 \text{ to } 1899.8 \quad (\text{x.})$$

$$\left. \begin{aligned} x &= 0''.008 \sin \odot - 0''.064 \cos \odot \\ y &= 0''.056 \sin \odot - 0''.006 \cos \odot \end{aligned} \right\} \text{from } 1900.0 \text{ to } 1907.0 \quad (\text{xi.})$$

From these the Kimura z -term is omitted.

The axis of x is from the north pole towards Greenwich, and that of y is 90° W. in most investigations on the variation of latitude. In this paper the axis of y is taken in the opposite direction, in order to make the axes right-handed. Thus

$$l = x \text{ and } m = -y \quad (\text{xii.})$$

Then

$$\left. \begin{aligned} \lambda &= 0''.006 \sin \odot - 0''.036 \cos \odot \\ \mu &= 0''.062 \sin \odot + 0''.009 \cos \odot \end{aligned} \right\} \text{from } 1893.8 \text{ to } 1899.8 \quad (\text{xiii.})$$

$$\left. \begin{aligned} \lambda &= 0''.015 \sin \odot + 0''.002 \cos \odot \\ \mu &= 0''.019 \sin \odot + 0''.015 \cos \odot \end{aligned} \right\} \text{from } 1900.0 \text{ to } 1907.0 \quad (\text{xiv.})$$

In both intervals the motion of the pole of rotation is direct, while in the first that of the pole of inertia is retrograde.

* *Astronomische Nachrichten*, Nr. 4344, Bd. 181.

The determinations of E. F. van de Sande Bakhuyzen * give

$$\left. \begin{aligned} x &= 0''.004 \sin \odot - 0''.123 \cos \odot \\ y &= 0''.061 \sin \odot + 0''.061 \cos \odot \end{aligned} \right\} \text{from 1890 to 1892.} \quad (\text{xv.})$$

$$\left. \begin{aligned} x &= 0''.012 \sin \odot - 0''.112 \cos \odot \\ y &= 0''.057 \sin \odot + 0''.008 \cos \odot \end{aligned} \right\} \text{from 1893 to 1896.} \quad (\text{xvi.})$$

From these we deduce

$$\left. \begin{aligned} \lambda &= -0''.068 \sin \odot - 0''.051 \cos \odot \\ \mu &= 0''.083 \sin \odot - 0''.057 \cos \odot \end{aligned} \right\} \text{from 1890 to 1892} \quad (\text{xvii.})$$

$$\left. \begin{aligned} \lambda &= -0''.055 \sin \odot - 0''.045 \cos \odot \\ \mu &= 0''.075 \sin \odot + 0''.006 \cos \odot \end{aligned} \right\} \text{from 1893 to 1896} \quad (\text{xviii.})$$

Here again the motion is retrograde, though the Bakhuyzen orbits differ considerably in form from each other. All the four determinations, however, agree in the large positive coefficient of $\sin \odot$ in μ .

The Values of the Products of Inertia.

We have still to express F and G, and hence λ and μ , in terms of the amounts of mass-transference.

Suppose that at any point on the earth's surface the annual variation in the mass per unit area is enough to make

the excess of the value at the vernal equinox over the autumnal value $= \sigma$,

the excess of the value at the summer solstice over the winter value $= \sigma'$.

Then we can take the mass per unit area to be

$$\frac{1}{2}\sigma \cos \odot + \frac{1}{2}\sigma' \sin \odot \quad . \quad . \quad . \quad (\text{xix.})$$

Now

$$\begin{aligned} F &= \iiint \rho yz d\tau \text{ through the volume} \\ &= \iint \left(\frac{1}{2}\sigma \cos \odot + \frac{1}{2}\sigma' \sin \odot \right) yz dS \text{ over the surface} \\ &= a^4 \int_0^\pi \int_0^{2\pi} \left(\frac{1}{2}\sigma \cos \odot + \frac{1}{2}\sigma' \sin \odot \right) \sin^2 \theta \cos \theta \sin \phi d\theta d\phi \quad (\text{xx.}) \end{aligned}$$

and

$$G = a^4 \int_0^\pi \int_0^{2\pi} \left(\frac{1}{2}\sigma \cos \odot + \frac{1}{2}\sigma' \sin \odot \right) \sin^2 \theta \cos \theta \cos \phi d\theta d\phi \quad (\text{xxi.})$$

where a is the radius of the earth.

These values of F and G require to be corrected for the effects of elastic yielding in the earth. If the variable part of the mass per unit area is m , let m be supposed expanded in spherical harmonics. Let one term in this expansion be $a \sin^2 \theta \cos \theta \sin \phi$, say. Then F has exactly the same

* *Archives Néerlandaises des Sciences*, 1899, série ii., tome ii., p. 11.

value as if m was actually equal to aS_2 . Now the strain due to any harmonic in m can be treated separately, and the radial displacement will be proportional to the same harmonic as the surface density. Hence no harmonic other than S_2 can cause a displacement that will contribute anything to F . All we need to find, then, is the strain in the earth due to a surface layer of density aS_2 . This acts in two ways. It produces a force vertically downwards of gaS_2 per unit area of the surface, and also produces an internal gravitation potential $V_1 = \frac{4}{3}\pi a/a r^2 S_2/a$. The solution of this problem in the case where the earth is supposed homogeneous and incompressible is given by Sir G. H. Darwin.* The internal potential in this case produces exactly the same strain as a radial stress $\frac{3}{5}gaS_2$ per unit area. In all, then, the internal strain is the same as if there were no normal force, and there were an internal disturbing potential $-\frac{2}{3}V_1$. The deformation due to this will cause an additional external potential $-\frac{2}{3}kV_0$, where V_0 is the external gravitational potential due to the distribution aS_2 , and k is a constant that depends on the rigidity. The term in S_2/r^3 in the external potential is proportional to F , by MacCullagh's theorem. Hence the effect of internal elasticity is that the uncorrected value of F must be multiplied by $(1 - \frac{2}{3}k)$ to give the true value. Similar arguments apply to G .

If heterogeneity is allowed for, the vertical depression at the surface due to the load must be greater on account of the smaller rigidity of the outer layers. This increase will, however, be largely counterbalanced by the fact that the density of the outer layers is also smaller.

Compressibility may cause a further increase. The value of k is known from the 14-monthly variation of latitude to be about $\frac{1}{4}$.† In all, the value of the correcting factor is likely to be about $\frac{3}{4}$. If the correcting factor is κ , $C_0 = \frac{1}{3}Ea^2$, and the mean density is 5.5, we have

$$\left. \begin{aligned} \lambda &= -9'' \cdot 1 \times 10^{-3} \kappa \int_0^\pi \int_0^{2\pi} (\sigma \cos \odot + \sigma' \sin \odot) \sin^2 \theta \cos \theta \cos \phi d\theta d\phi \\ \mu &= -9'' \cdot 1 \times 10^{-3} \kappa \int_0^\pi \int_0^{2\pi} (\sigma \cos \odot + \sigma' \sin \odot) \sin^2 \theta \cos \theta \sin \phi d\theta d\phi \end{aligned} \right\} \text{(xxii.)}$$

where σ and σ' must now be expressed in grams per square centimetre. It will be found convenient, on account of the uncertainty in the value of κ , to put $\lambda_0 = \lambda/\kappa$, $\mu_0 = \mu/\kappa$.

We are now in a position to treat various putative causes separately.

There are several obvious ways in which seasonal variation in temperature can influence the products of inertia. Evidently, if the whole of the surface was of similar character, either land or sea, or even if the distribution of land and sea was symmetrical

* *Scientific Papers*, vol. ii. p. 8.

† Sir Joseph Larmor, *Proc. Lond. Math. Soc.*, series 2, vol. xiv., 1915, p. 449.

about the polar axis, F and G would be necessarily zero, and the only possible effect on the latitude would be a Kimura z -term, much too small to be detected. The existence of an annual variation of latitude must then be attributed in some way to the differences in properties between land and sea. The methods of mass-transference that seem to require consideration first are as follows:—

- (1) Periodical convection currents in the atmosphere due to unequal heating of land and water.
- (2) Periodical convection currents in the ocean.
- (3) The formation of ice round the poles.
- (4) Precipitation of both rain and snow.
- (5) Periodical changes in vegetation, such as the formation of deciduous parts of trees, the rise of sap in them, and the formation of annual parts of herbs.

II. *The Effect of Atmospheric Motion.*

It is known that in summer areas of high barometric pressure tend to form over the oceans, and areas of low pressure over the continents. This fact has led R. Spitaler * to investigate the effect of atmospheric disturbance on the rotation of the earth. His treatment, while suggestive as a first treatment of the problem, seems open to improvement in several respects. In the first place, he assumes that the amount of variation in the mass of air above a particular square centimetre of the earth's surface can be obtained by simply dividing the variation in barometric pressure by the value of gravity. This appears a natural hypothesis at first sight, but as it involves the neglect of both inertia and viscosity, its accuracy is not very obvious. I have, however, examined the effect of atmospheric convection currents in certain particular cases, allowing for the increase in the apparent viscosity due to eddy motion,† and find that the hypothesis is justified with some accuracy in the cases I have considered. Secondly, he confines his attention to the differences in barometric pressure between January and July, omitting those between April and October, which are not negligible. Thirdly, he takes the number τ to be 305, in place of 430. As, however, he only considers the motion of the pole of inertia, without finding that of the pole of rotation from it, the fact that this gives the wrong sign for $n'^2 - n^2/\tau^2$ does not lead to serious consequences. Lastly, the method of combining the observations is questionable. The method is to treat each continent and ocean separately, to compute the average pressure difference over it, and to replace the actual pressure variation at every point by this average value. Thus the resultant of the pressure variation over each area is assumed to be completely replaceable by a thrust at the centroid of the area. This is not obviously true, and it seems more satisfactory to find the effects by a direct numerical integration.

In the present investigation I shall adopt Spitaler's first

* *Denkschriften d. Akad. d. Wiss. z. Wien*, Bd. 64, 1897.

† G. I. Taylor, *Phil. Trans.*, 1915, vol. ccxv., A, pp. 1-26.

hypothesis, but not the others. The numerical data have been taken from Bartholomew's Meteorological Atlas, plate 12.

In the following tables the pressure variations are referred to the unit $0''.01$ of mercury. The mass of a column of mercury 1 cm. square and $\frac{1}{100}$ of an inch in height is 0.34 gram, so that to convert these numbers to grams per square centimetre the factor 0.34 must be applied.

If at any place the excess of pressure in January over that in July is h , and the excess of pressure in April over that in October is h' , and the sun's longitude on April 15 is α , then the annual part of the pressure is $-\frac{1}{2}h \sin (\odot - \alpha) + \frac{1}{2}h' \cos (\odot - \alpha)$, provided that among the periodic terms occurring there is none whose period is an odd submultiple of a year.

The value of the mass per unit area is therefore

$$0.34(\frac{1}{2}h' \cos \alpha + \frac{1}{2}h \sin \alpha) \cos \odot + 0.34(\frac{1}{2}h' \sin \alpha - \frac{1}{2}h \cos \alpha) \sin \odot.$$

$$\begin{aligned} \text{Thus} \quad \sigma &= 0.34(h' \cos \alpha + h \sin \alpha) \} \\ \sigma' &= -0.34(h \cos \alpha - h' \sin \alpha) \} \quad \cdot \quad \cdot \quad \cdot \quad (i.) \end{aligned}$$

These values of σ and σ' must be substituted in the equations to find λ and μ . α is taken to be $24^\circ 24'$.

In the tables no entries are made for the parallel of 40° south and all further south. The reason is seen at once on inspection of the isobars. In the area in question all of them are practically small circles of constant latitude, so that the variation of pressure from month to month along each must be nearly uniform. The values of the four integrals computed can therefore receive no important part from this southern area, on account of its symmetry.

We find then

$$\begin{aligned} \int_0^\pi \int_0^{2\pi} h \sin^2 \theta \cos \theta \sin \phi d\theta d\phi &= 28.3 \\ \int_0^\pi \int_0^{2\pi} h \sin^2 \theta \cos \theta \cos \phi d\theta d\phi &= 4.8 \\ \int_0^\pi \int_0^{2\pi} h' \sin^2 \theta \cos \theta \sin \phi d\theta d\phi &= -5.68 \\ \int_0^\pi \int_0^{2\pi} h' \sin^2 \theta \cos \theta \cos \phi d\theta d\phi &= -7.10 \end{aligned}$$

Substituting these in the equations already obtained (II. i. and I. xxii.), we get

$$\begin{aligned} \lambda_0 &= 0''.0138 \cos \odot + 0''.0226 \sin \odot \} \\ \mu_0 &= -0''.0202 \cos \odot + 0''.0866 \sin \odot \} \quad \cdot \quad \cdot \quad (ii.) \end{aligned}$$

Comparing these with the values derived from observation, we observe—

- (1) The theoretical motion is direct instead of retrograde.
- (2) The largest coefficient in both is the coefficient of $\sin \odot$ in μ .

Pressure Variations h = January Pressure - July Pressure.

$\frac{\phi}{\theta}$	0.	20.	40.	60.	80.	100.	120.	140.	160.	180.	200.	220.	240.	260.	280.	300.	320.	340.	$\Delta h \sin \phi$	$\Delta h \cos \phi$	$\sin^2 \theta \cos \phi$	$\sin^2 \theta \sin \phi$	$\sin^2 \theta \cos \theta$	$\sin^2 \theta \sin \theta$
0	?																		?	?	0.000	0	0	0
10	-3	-10*	-9	-9	2	13	16	17	14	11	9	9	6	6*	3*	-1*	-1	0	7 - 91	?	.030	0	-3	3
20	-25	-28	-15	0	18*	30*	36*	36*	28*	24	12*	10	13	10	5	-10	-14*	-20	81 - 242	?	.110	9	-27	9
30	-10	+5	19*	29*	50*	68*	73*	36*	9	-13	-9*	-9*	23*	21*	10	-6	-32	12	231 - 65	?	.216	50	-14	27
40	4	18*	33*	51*	75*	91*	75*	19*	-20	-23	-35	-35	10*	32*	15*	-6	-19	-11	315 + 84	?	.316	99	40	40
50	15	15*	33*	57*	87*	96*	61*	19*	-15	-33	-46	-32	25*	35*	16*	2	-19	-16	296 + 105	?	.377	116	10	10
60	13*	16*	37*	67*	63*	75*	66*	33	7	-9	-4	-12	10	19*	8	-1	-12	-2	286 + 26	?	.375	107	10	10
70	10*	16*	34	49	38*	33*	33	17	4	-2	2	7	7	10*	2	0	-5	3	158. 31	?	.302	48	9	9
80	-2*	2*	14*	20	17	13	7	3	1	-3	-5	-4	-3	0	2	-3	-4	-3	77 20	?	.168	13	3	3
90	-10	-14*	-11*	-6	-4	-3*	-5*	-6	0	-2	-5	-5	-8	-8	-3	-5*	-5	-8	4 - 26	?	.000	0	0	0
100	-14	-10*	-20*	-13	-14	-16	-20*	-20	-9	-9	-10	-10	-15	-18	-9	-16*	-19*	-13	-11 - 11	?	.168	2	2	2
110	-14	-26*	-30	-24	-23	-17	-30*	-27*	-14	-11	-10	-11	-10	-11	-12	-27*	-26	-14	-50 - 39*	?	.302	15	12	12
120	-6	-23*	-22	-26	-20	-11	-25*	-29*	-15	-4	-4	4	2	-9	-3	-23*	-16	-11	-80 - 39	?	.375	30	15	15

An asterisk indicates that the corresponding point is on land.

Pressure Variations h' April October.

ϕ θ	0.	20.	40.	60.	80.	100.	120.	140.	160.	180.	200.	220.	240.	260.	280.	300.	320.	340.
0	?				?												?	
10	12	12*	12	8	11	17	22	20	20	18	15	13	12	13*	11*	21*	18	13
20	25	25	14	10	11*	15*	15*	9*	8*	11	10*	6	9	12	19	20	24*	20
30	4	6	-2*	0*	3*	2*	-8*	2*	7	12	20*	10*	5*	15*	12	13	17	14
40	-3	-11*	-14*	-12*	-9*	-10*	-15*	-6*	5	10	4	0	-1*	5*	2*	-3	-9	-7
50	-14	-22*	-11*	-21*	-21*	-19*	-14*	-7*	13	14	5	20	-10*	-10*	-12*	-15	-14	-11
60	-5*	-9*	-2*	-10*	-18*	-14*	-10*	+2	18	20	23	21	3	-7*	-1	-5	1	-4
70	-4*	-1*	0	-7	-10*	-13*	0	6	13	14	11	8	9	-1*	4	11	4	-1
80	-2*	-1*	-1*	-1	-2	-7	-3	6	10*	3	7	9	5	1	0	2	2	0
90	-7	-4*	-4*	-8	-5	-14*	-4*	-1	5	11	7	7	2	-9	1	1*	0	-4
100	-9	-1*	-3*	-8	-10	-6	-10*	-6	0	4	5	3	0	-4	-5	0*	-2*	-7
110	-6	0*	3	-9	-14	-9	-2*	-2*	-4	-3	0	-3	-1	-15	-10	2*	0	-7
120	-1	5*	-4	-1	-20	-25	-1*	7*	2	-9	-23	-26	-15	-19	-14	-1*	-1	-8

An asterisk indicates that the corresponding point is on land.

The theoretical amplitude is greater than the observational one, although the amount of the excess varies greatly with the particular reduction employed for the latter.

Spitaler's amplitude is $0''.1063$, which would have to be multiplied by a further factor $\frac{4.30}{3.0}$ to correspond to mine. The difference is probably due to the different methods of integration. As the continents are so large, there is a tendency for the effects of extra masses at opposite sides of a continent to displace the pole of inertia, not in the same direction, as would be the case if Spitaler's treatment were correct, but sometimes in very different directions, so that the algebraic sum of the displacements may be much less than the value obtained by Spitaler, which is practically an arithmetical sum. His method similarly tends to exaggerate the importance of the great seasonal changes in high latitudes. We should, then, expect *a priori* that his amplitude would be greater than mine.

III. *Correction for Height above Sea-level.*

In the discussion following the reading of this paper the necessity of applying a correction to the values found for the displacements of the pole by atmospheric agencies was pointed out by Mr. W. W. Bryant. The pressures given in the meteorological charts used are not the true atmospheric pressures at the places concerned, but values reduced to sea-level according to the formula given by Laplace.* For our purpose, however, we need not these modified values, but the variations in the true local pressures; the correction included in the charts therefore needs to be taken out again.

With sufficient accuracy Laplace's formula may be written

$$z = 1.8 \times 10^6 \left[1 + \frac{2(t+t_0)}{1000} \right] \left[\log_{10} \frac{p_0}{p} \right]$$

where z is the height above sea-level, measured in centimetres.

t is the temperature at the point concerned.

t_0 is the corresponding temperature at sea-level.

p is the pressure at the point concerned.

p_0 is the corresponding pressure at sea-level.

The charts give p_0 , but what we need is p . If $\delta\chi$ denote "the periodic part of χ " we have

$$p = p_0 \text{ antilog } \left\{ - \frac{z}{1.8 \times 10^6} \left(1 - \frac{t+t_0}{500} \right) \right\},$$

and hence

$$\begin{aligned} \delta p = \delta p_0 \text{ antilog } & \left\{ - \frac{z}{1.8 \times 10^6} \left(1 - \frac{t+t_0}{500} \right) \right\} \\ & + p_0 z \frac{\log_e 10}{1.8 \times 10^6} \frac{\delta(t+t_0)}{500} \text{ antilog } \left\{ - \frac{z}{1.8 \times 10^6} \left(1 - \frac{t+t_0}{500} \right) \right\}. \end{aligned}$$

Write $1.8 \times 10^6 = 1/\alpha$.

* *Mécanique Céleste*, tome iv., p. 294, edition of 1880.

In the reduction to sea-level the value of $t - t_0$ employed was invariable, so that $\delta t = \delta t_0$.

The correction we must apply is then

$$\delta p - \delta p_0 = -\delta p_0(1 - 10^{-az}) + 9.2 \times 10^{-3} p_0 a z 10^{-az} \delta t \quad (i.)$$

where t is measured in degrees centigrade.

In the case of the Asiatic plateau, which contributes most to the motion, the average height is about 8×10^4 cm., and hence $az = 4.3 \times 10^{-2}$ roughly. This would make the correction equal to

$$-0.092 \delta p_0 + 1.07 \times 10^{-2} \delta t.$$

The two parts of this act in the same direction, but the second part is about five times as great as the first. In all, the effect of this correction is enough to reduce the local-pressure variation to about half of the amount that has been assumed above.

Using the formula (i.) and the heights given in my table,* I find that the correction required is

$$\lambda_0 = 0''.0000 \cos \odot - 0''.0014 \sin \odot,$$

$$\mu_0 = 0''.0128 \cos \odot - 0''.0315 \sin \odot.$$

IV. The Movement of the Ocean.

There are several causes that tend to produce periodic movement in the ocean. Such are

(1) The inflow of water to the sea. This is a local phenomenon, and will not be considered here.

(2) Evaporation, which may produce a periodic change in the level. It would, however, be most conveniently treated as a correction to the theory of precipitation.

(3) Temperature variations, which may produce periodical convection currents, resulting in variations of mass per unit area.

(4) The periodic alteration in atmospheric pressure, which must react on the sea.

In this section the third and fourth agencies will be treated.

With sufficient accuracy for our purpose we can suppose that all the radiation supplied to the ocean is absorbed at the free surface. It is well known that a very small thickness of water is required to stop the dark heat radiation: light is more penetrating, but H. Wild gives † for distilled water and white light the transmission coefficient 0.98835, the unit of length being one Parisian inch; so that after passing through a layer of distilled water 15 metres thick the intensity is reduced to about $\frac{1}{500}$ of its amount at the surface. Sea water would probably absorb it still more rapidly. Some of the ultra-violet radiation is more penetrating, ‡ but it is smaller in amount than either of the other two types,

* *Memoirs of the R. A. S.*, vol. lx., 1915, p. 191.

† *Poggendorff's Annalen*, vol. xcix., 1856, p. 272.

‡ Murray, *The Depths of the Ocean*, pp. 250-253.

and will here be omitted. As annual temperature oscillations in the ocean are known to penetrate to a depth of some hundreds of feet, this step seems justified.

The depth of the ocean is a small fraction of the radius of the earth, so that no considerable error will be introduced if we treat a case of a plane boundary instead of a spherical one. Take the origin in the mean position of the free surface. The axis of z is vertically upwards, while those of x and y lie along the mean free surface. If the temperature is V , the transmission of heat is governed by the equation $\frac{DV}{Dt} = k\nabla^2 V$, where k is a kind of coefficient of thermometric conductivity. It is not equal to the ordinary coefficient, since it contains a very large part due to eddy motion, which must produce in the ocean an effect similar to that found by G. I. Taylor* for the atmosphere. Neglecting second-order terms, we may replace D/Dt by $\partial/\partial t$.

Now all the velocities and also the temperature and pressure may change very rapidly in regard to z , but much less rapidly in regard to x and y . Hence in a first approximation we need only consider

$$\frac{\partial V}{\partial t} = k \frac{\partial^2 V}{\partial z^2} \quad . \quad . \quad . \quad . \quad (i.)$$

If the surface temperature is $Fe^{\gamma t}$, where F is a function of x and y only, this gives

$$V = Ae^{\gamma t} \exp. \sqrt{\frac{\gamma}{2k}}(1+i)z + Be^{\gamma t} \exp. -\sqrt{\frac{\gamma}{2k}}(1+i)z. \quad (ii.)$$

where A and B are at present arbitrary, save that $A+B=F$.

The observations given by Murray indicate that the temperature oscillation lags more and more as the depth increases, and extends downwards in all through some hundreds of feet. It behaves, indeed, like the first term of this value of V , and as the depth of the ocean is much greater than the vertical range of the temperature oscillation, it follows that $B=0$, and therefore $A=F$.

Observations† by the *Michael Sars* in the North Atlantic outside the Sognefjord have shown that while the temperature at the surface varies through 9°C ., the variation is exactly reversed in phase at 200 metres, and the range is reduced to 1.2°C . This observation indicates that $\sqrt{(2k/\gamma)}$ for a yearly variation is of the order of 60 metres. As $\gamma = 1.99 \times 10^{-7}/1 \text{ sec.}$, we see that $k = 3.58 \text{ cm.}^2/\text{sec.}$ The part of it due to ordinary conductivity is only about 0.0012 ‡ in the same units, so that practically the whole of this apparent conductivity is due to local convection currents and eddy motion. Now G. I. Taylor has shown that the apparent kinematic coefficient of viscosity due to eddy motion is equal to the apparent coefficient of thermometric conductivity due to the same cause, and consequently in the equations of motion of

* *Phil. Trans.*, 1915, *loc. cit.*

† Murray, *ibid.*, p. 228.

‡ See Ganot, *Physics*, p. 408.

a viscous fluid we can put for it the value k . The ordinary viscosity is sufficiently small to be swamped by this part.

The equations of motion are then

$$\frac{Du}{Dt} = X - \frac{1}{\rho} \frac{\partial p}{\partial x} + k \nabla^2 u + \frac{1}{3} k \frac{\partial \delta}{\partial x}, \text{ etc.} \quad (\text{iii.})$$

$$\text{where} \quad \delta \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial \rho}{\partial t} \quad (\text{iv.})$$

The law of volumetric expansion gives, further,

$$\rho = \rho_0 (1 - \alpha V) \quad (\text{v.})$$

where α is a small constant.

In the undisturbed state $X = Y = 0$, $Z = -g$, and hence the undisturbed pressure $p_0 = -g\rho_0 z$. Put then $p = p_0 + p'$, where p' is a small quantity. Let the elevation of the free surface be ξ . Then the equations of motion reduce to

$$\left. \begin{aligned} \frac{\partial u}{\partial t} &= - \frac{1}{\rho_0} \frac{\partial p'}{\partial x} + k \nabla^2 u + \frac{1}{3} k \frac{\partial \delta}{\partial x} \\ \frac{\partial v}{\partial t} &= - \frac{1}{\rho_0} \frac{\partial p'}{\partial y} + k \nabla^2 v + \frac{1}{3} k \frac{\partial \delta}{\partial y} \\ \frac{\partial w}{\partial t} &= g \alpha V - \frac{1}{\rho_0} \frac{\partial p'}{\partial z} + k \nabla^2 w + \frac{1}{3} k \frac{\partial \delta}{\partial z} \end{aligned} \right\} \quad (\text{vi.})$$

$$\text{with} \quad \delta = \alpha \frac{\partial V}{\partial t} \quad (\text{vii.})$$

In the last of these the terms involving the motions may be omitted. If then $g\rho_0 \xi'$ be the value of p' when $z = 0$, we have

$$\begin{aligned} p' - g\rho_0 \xi' &= g\rho_0 \alpha \int_0^z V dz \\ &= g\rho_0 \alpha \frac{F}{\lambda} (e^{\lambda z} - 1) e^{\nu t} \end{aligned} \quad (\text{viii.})$$

$$\text{where} \quad \lambda = \sqrt{\frac{\gamma}{z k}} (1 + \iota).$$

The first two equations give

$$\left. \begin{aligned} \frac{\partial u}{\partial t} - k \nabla^2 u &= - \frac{\partial}{\partial x} \left\{ g \xi' + \frac{g \alpha}{\lambda} F (e^{\lambda z} - 1) e^{\nu t} \right\} \\ \frac{\partial v}{\partial t} - k \nabla^2 v &= - \frac{\partial}{\partial y} \left\{ g \xi' + \frac{g \alpha}{\lambda} F (e^{\lambda z} - 1) e^{\nu t} \right\} \end{aligned} \right\} \quad (\text{ix.})$$

Then

$$\left. \begin{aligned} u &= - \frac{1}{\iota \gamma} \frac{\partial}{\partial x} \left(g \xi' - \frac{F g \alpha}{\lambda} e^{\nu t} \right) + \frac{\partial}{\partial x} \left\{ \frac{g \alpha}{\lambda} \frac{F z e^{\lambda z} e^{\nu t}}{2 k \lambda} + C e^{\lambda z} e^{\nu t} + C' e^{-\lambda z} e^{\nu t} \right\} \\ v &= - \frac{1}{\iota \gamma} \frac{\partial}{\partial y} \left(g \xi' - \frac{F g \alpha}{\lambda} e^{\nu t} \right) + \frac{\partial}{\partial y} \left\{ \frac{g \alpha}{\lambda} \frac{F z e^{\lambda z} e^{\nu t}}{2 k \lambda} + D e^{\lambda z} e^{\nu t} + D' e^{-\lambda z} e^{\nu t} \right\} \end{aligned} \right\} \quad (\text{x.})$$

C, C', D, D' being functions of x and y only.

As the depth is great, C' and D' must be small. The only purpose of including them is to make it possible to satisfy the conditions $u=0$ and $v=0$ at the bottom. Except within a small distance from the bottom their effect is necessarily insensible. The terms depending on C and D are due to reflected waves from the margins. In the case of an ocean of infinite extent (corresponding to the case of an ocean covering the whole earth) they would be zero.

Now one of the surface conditions is that p_{zz} is continuous there. The atmospheric pressure being Π , we have then

$$-\Pi = (p_{zz})_{z=\zeta} = \left[-p - \frac{2}{3}k\rho_0\delta + 2k\rho_0\frac{\partial w}{\partial z} \right]_{z=\zeta} \\ = g\rho_0\zeta + \frac{4}{3}k\rho_0\alpha\gamma Fe^{\gamma t}g\rho_0\zeta' \quad . \quad . \quad (xi.)$$

This determines ζ' in terms of ζ .

Now

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\gamma} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \left(g\zeta' - \frac{Fga}{\lambda} e^{\gamma t} \right) \\ + \frac{ga}{2k\lambda^2} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) Fze^{\lambda z} e^{\gamma t} + \left(\frac{\partial C}{\partial x} + \frac{\partial D}{\partial y} \right) e^{\lambda z} e^{\gamma t} + \left(\frac{\partial C'}{\partial x} + \frac{\partial D'}{\partial y} \right) e^{-\lambda z} e^{\gamma t} \\ = -\frac{1}{\gamma} \nabla_1^2 \left(g\zeta + \frac{\Pi}{\rho} + \frac{4}{3}k\alpha\gamma Fe^{\gamma t} - \frac{Fga}{\lambda} e^{\gamma t} \right) \\ + \frac{ga}{2\gamma} \nabla_1^2 Fze^{\lambda z} e^{\gamma t} + \left(\frac{\partial C}{\partial x} + \frac{\partial D}{\partial y} \right) e^{\lambda z} e^{\gamma t} + \left(\frac{\partial C'}{\partial x} + \frac{\partial D'}{\partial y} \right) e^{-\lambda z} e^{\gamma t} \quad (xii.)$$

If now h be the depth of the ocean and m the variable part of the mass of the vertical column of water above unit area, we have

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial t} \int_{-h}^{\zeta} \rho dz \\ = \rho_0 \frac{\partial \zeta}{\partial t} - \int_{-h}^0 \rho_0 \alpha \gamma F e^{\lambda z} e^{\gamma t} dz \\ = \rho_0 \gamma \zeta - \frac{\rho_0 \alpha \gamma}{\lambda} F e^{\gamma t} \quad . \quad . \quad (xiii.)$$

Also, since the rate of change of the mass in any column is equal to the rate at which matter enters that column,

$$\frac{\partial m}{\partial t} = - \int_{-h}^{\zeta} \rho \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz \\ = \frac{\rho_0 h}{\gamma} \nabla_1^2 \left(g\zeta + \frac{\Pi}{\rho} + \frac{4}{3}k\alpha\gamma Fe^{\gamma t} - \frac{Fga}{\lambda} e^{\gamma t} \right) \\ - \frac{\rho_0 g a}{2\gamma} \frac{1}{\lambda^2} \nabla_1^2 F e^{\gamma t} - \frac{\rho_0}{\lambda} \left(\frac{\partial C}{\partial x} + \frac{\partial D}{\partial y} \right) e^{\gamma t} + \frac{\rho_0}{\lambda} \left(\frac{\partial C'}{\partial x} + \frac{\partial D'}{\partial y} \right) e^{\gamma t} \quad . \quad (xiv.)$$

Equating these two values we have for ζ the equation

$$\left(\frac{gh}{\gamma^2}\nabla_1^2 + 1\right)\left(\zeta - \frac{Fa}{\lambda}e^{\gamma t}\right) = -\frac{h}{\gamma^2}\nabla_1^2\left(\frac{\Pi}{\rho} + \frac{4ka\gamma}{3}Fe^{\gamma t}\right) - \frac{ga}{2}k\gamma\nabla_1^2Fe^{\gamma t} - \frac{1}{\gamma\lambda}\left(\frac{\partial(C-C')}{\partial x} + \frac{\partial(D-D')}{\partial y}\right)e^{\gamma t} \quad (\text{xv.})$$

Now in the actual case of the movement of the ocean the functions like ζ , F , and Π that we are concerned with are of such a form that they are practically periodic functions of position, with a wave-length equal to the circumference of the earth. Thus if K denotes any one of them, $\frac{\partial^2 K}{\partial x^2} + \frac{\partial^2 K}{\partial y^2}$ is practically $-\frac{2K}{a^2}$, where a is the radius of the earth.

Then $\frac{gh}{\gamma^2}\nabla_1^2$ is of order 10^5 , so that the term 1 in the same bracket can be dropped. Again, the term depending on the complementary functions is seen to be of the same order as $g\zeta/\gamma^2\lambda a^2$, which is of order 500ζ . It can therefore be neglected in comparison with $10^5\zeta$.

It is obvious that a uniform pressure or temperature variation over the surface could produce no translation. Hence m depends only on the excesses of Π and F over their mean values over the ocean surface. Let these excesses be Π' , F' .

The equation can thus be reduced to the form $\nabla_1^2\zeta = \nabla_1^2\chi$, and can be integrated in the form

$$\zeta - \frac{Fa}{\lambda}e^{\gamma t} + \frac{\Pi}{\rho g} + \frac{4}{3}\frac{ka\gamma}{g}Fe^{\gamma t} + \frac{k\gamma^3 a}{2h}Fe^{\gamma t} = \Theta(x, y) \quad (\text{xvi.})$$

where $\Theta(x, y)$ is a function satisfying $\nabla_1^2\Theta = 0$.

Next, if m_0 be the mean value of m at any place we have

$$\begin{aligned} m - m_0 &= \rho_0\left(\zeta - \frac{Fa}{\lambda}e^{\gamma t}\right) \\ &= -\frac{\Pi}{g} - \frac{4}{3}\frac{k\rho_0 a\gamma}{g}Fe^{\gamma t} - \frac{k\rho_0 \gamma^3 a}{2h}Fe^{\gamma t} + \rho_0\Theta \quad (\text{xvii.}) \end{aligned}$$

As the total value of $\iint (m - m_0)dS$ over the surface of the ocean must be zero, we can replace Π by Π' , F by F' , and Θ by Θ' , where $\iint (\Theta - \Theta')dS$ over the surface is now 0.

The term in Θ' arises through viscous resistance at the boundaries to vertical motion. In mid-ocean it may be expected to be insensible. The terms "on the right" involving F as a factor are also utterly negligible, being of the order of 10^{-12} grams per square centimetre. It is further easily verified that the error arising from the neglect of w in the third equation of motion is very small.

$$\text{Hence} \quad m - m_0 = -\Pi'/g \quad . \quad . \quad . \quad (\text{xviii.})$$

But the variable part of the mass of air per unit area is equal to Π/g .

Hence the variable part of the mass of water per unit area and the variable part of the mass of air per unit area are together equal to $(\Pi - \Pi')/g$.

But $\Pi \approx \Pi'$ is the mean atmospheric pressure over the ocean surface. Call this Π_m . We therefore have the theorem that when water and air are taken together the mass per unit area is always uniform over the whole ocean surface. In other words, it is a function of the time alone, not of the position.

If m is the variable part of the mass per unit area over any point of the earth's surface, we have from the law of indestructibility of matter that $\iint m dS = 0$ over the earth's surface. Also m is constant over the ocean, equal to m' , say.

$$\text{Hence} \quad \left[m' \iint dS \right]_{\text{sea}} = - \iint m dS \text{ over the land.}$$

If, then, m_1 be the mean value of m over the land,

$$\begin{aligned} m' &= -m_1(\text{area of land})/(\text{area of sea}) \\ &= -.419m_1. \end{aligned}$$

$$\begin{aligned} \text{Next,} \quad F &= \iint myz dS \text{ over the whole surface} \\ &= m' \iint yz dS \text{ over the ocean} \\ &\quad + \iint myz dS \text{ over the land} \\ &= +.419m_1 \iint yz dS + \iint myz dS \quad . \quad . \quad (\text{xix.}) \end{aligned}$$

both taken over the land, since

$$\iint yz dS \text{ over the whole surface is necessarily } 0.$$

A large correction must therefore be applied to the values already found for the co-ordinates of the pole of inertia.

The tables already given are found to lead to the result that the annual part of m_1 is $-8.94 \sin(\odot - \alpha) - 2.08 \cos(\odot - \alpha)$. The corrected values of h and h' are accordingly found if we add to all the values of h given for land the number 7.5, and to all those of h' the number 2, and put all the oceanic values zero.

A repetition of the numerical double integration, omitting the correction of Section III., on these lines gives for the effects of atmosphere and ocean together the results

$$\begin{aligned}\iint h \sin^2 \theta \cos \theta \sin \phi d\theta d\phi &= 22.9 \\ \iint h \sin^2 \theta \cos \theta \cos \phi d\theta d\phi &= 0.96 \\ \iint h' \sin^2 \theta \cos \theta \sin \phi d\theta d\phi &= -5.23 \\ \iint h' \sin^2 \theta \cos \theta \cos \phi d\theta d\phi &= -1.84.\end{aligned}$$

Whence

$$\left. \begin{aligned}\lambda_0 &= 0''.0040 \cos \odot + 0''.0051 \sin \odot \\ \mu_0 &= -0''.0134 \cos \odot + 0''.0659 \sin \odot\end{aligned} \right\} \quad (\text{xx.})$$

The motion is of the same type as that found in Chapter II., but the amplitude of the largest term is reduced by 20 per cent., while the others are still further reduced. The ellipse described is thus much more eccentric than the previous one.

In the above treatment the rotation of the earth has not been included, and its effect will certainly be great. Thus Lamb gives* as the equation for the elevation of the free surface of a rotating plane ocean under a gravitational field

$$\nabla_1^2 \zeta + \frac{\gamma^2 - 4\omega^2}{gh} \zeta = \nabla_1^2 \zeta' \quad \cdot \quad \cdot \quad \cdot \quad (\text{xxi.})$$

where ζ' is the equilibrium height.

If we omit ω , the equation reduces to $\zeta = \zeta'$ for long-period tides, so that the tide has the equilibrium height, as has been already shown. Including it, however, writing $\nabla_1^2 = -2/a^2$, and taking the depth of the ocean to be 3×10^5 cm., we see that the second term on the left is about ten times the first, so that the amplitude is reduced from ζ' to $\zeta'/11$. Thus it may be expected that on account of rotation the correction applied in this chapter for the oceanic motion will have to be reduced very considerably. The true motions of the pole of inertia will then lie between the estimates made in this chapter and the last, but presumably nearer the earlier estimate.

The reduction from the equilibrium theory here indicated agrees roughly with that given by Dr. G. R. Goldsbrough† for the case of long-period tides in a zonal ocean. His correcting factors range from 0.08 to 0.2. The best possible estimate at present is then to be obtained by taking the motion of the pole of inertia uncorrected for the ocean, and applying $\frac{1}{10}$ of the correction that has been given in this chapter.

$$\left. \begin{aligned}\text{Thus } \lambda_0 &= 0''.0128 \cos \odot + 0''.0219 \sin \odot \\ \mu_0 &= -0''.0195 \cos \odot + 0''.0845 \sin \odot\end{aligned} \right\} \quad (\text{xxii.})$$

These co-ordinates now include the effects of both atmospheric and oceanic movements.

* *Hydrodynamics*, p. 303, equation (12).

† *Proc. Lond. Math. Soc.*, 1915, vol. xiv. p. 223.

Adding the correction of Chapter III. now, we have

$$\left. \begin{aligned} \lambda_0 &= 0''.0128 \cos \odot + 0''.0203 \sin \odot \\ \mu_0 &= -0''.0067 \cos \odot + 0''.0530 \sin \odot \end{aligned} \right\} \quad . \quad (\text{xxiii.})$$

V. *The Effect of Precipitation.*

In this investigation we are concerned with only those modes of transport of mass that cause a variation in the amount of matter per unit area of the surface. For instance, if rain falls upon a particular spot, and immediately runs away to the sea, then it contributes nothing to the elements of the products of inertia corresponding to that point. Similarly, if it evaporates at once, it contributes nothing to the elements derived from terrestrial data; the presence of water vapour in the atmosphere will, however, affect the weight of the atmosphere locally, and consequently its mass will be included in the variations of distribution of atmospheric mass already considered. Thus the effect of water vapour in the atmosphere is already included in the treatment of Section II. This section is concerned only with those parts of the precipitation that seem to remain on the ground for some considerable time. These are, first, the water that is absorbed by the soil; second, the snow that lies on the ground in winter. Data for the first part seem to be almost entirely lacking. In this country the soil is wetter in winter than in summer, principally owing to diminished evaporation, as the rainfall does not seem to vary to such an extent as the soil-humidity. In countries, however, where most of the precipitation falls in summer, this state of affairs may be reversed. On the whole it seems probable that the effect of retained soil-moisture is small in comparison with that of snowfall. Some estimate of the effect of snowfall is easily made. Snow accumulates in winter to a considerable depth in Canada and much of the United States, and over the greater part of Asia down to about the latitude of the Thian Shan. In South America the amount of snowfall seems to be small, while in Australia it is quite negligible. I have not succeeded in getting exact data for the amount of snow that accumulates in the northern hemisphere during winter, but rough estimates can be made. For instance, in the map in Bartholomew's *Meteorological Atlas** that shows the amount of precipitation in the autumn and winter quarters, it is seen that in Eurasia it is greatest in Russia in Europe, and steadily diminishes eastwards, ceasing to be measurable about longitude 100° E., corresponding roughly to Lake Baikal. A reason for this is seen in the fact that the prevailing wind in Northern Asia in winter blows from the west, so that the further east it goes the drier the air may be expected to become. Again, Bartholomew's monthly isothermals† for the world show that most of Siberia is below 0° C. from October till March or April, so that any precipitation in the winter half

* Plate 21, *loc. cit.*

† Plate 3, *loc. cit.*

of the year will be in solid form. For more satisfactory data I must express my thanks to Sir W. N. Shaw, who has kindly informed me that snow in Siberia commences to accumulate in October, and reaches its maximum depth in March; this maximum depth averages nearly 1 metre. This covering disappears by the end of May. It is usual to regard 10 cm. of newly fallen snow as equivalent to 1 cm. of rain, but as this Siberian snow becomes more and more compressed under its own weight during the winter, a smaller divisor, say 3, must be employed to reduce it to the equivalent amount of water. The area of Europe and Asia that is covered with snow in winter is then included in the part whose east longitude is less than 100° , and whose co-latitude is not more than 40° .

The greatest and most extensive snowfall in North America seems to be on the Atlantic side; this fact is presumably connected with the north-westerly trend of the winter isotherms there. The greatest snowfall in the United States is near Lake Superior (long. 85° W.), where a fall of 130 inches is attained,* corresponding to about 50 cm. of rain.

If m is the mass of snow per square centimetre of the ground at any instant, we require the annual part of it. Let this be as before $\frac{1}{2}\sigma \cos \odot + \frac{1}{2}\sigma' \sin \odot$. Further, suppose the amount to increase uniformly from the September equinox till the March equinox, and then to diminish uniformly till $\odot = 2\pi + \frac{1}{3}\pi$. If the maximum depth is ϖ , we have

$$m = \frac{\varpi}{\pi}(\odot - \pi) \text{ from } \odot = \pi \text{ to } \odot = 2\pi$$

$$m = \varpi(\frac{1}{3}\pi - \odot) \div \frac{1}{3}\pi \text{ from } \odot = 0 \text{ to } \odot = \frac{1}{3}\pi$$

$$m = 0 \text{ from } \odot = \frac{1}{3}\pi \text{ to } \odot = \pi.$$

$$\text{Then } \sigma = \frac{2}{\pi} \int_0^{2\pi} m \cos \odot d\odot = \frac{7\varpi}{\pi^2}$$

$$\sigma' = \frac{2}{\pi} \int_0^{2\pi} m \sin \odot d\odot = -\frac{3\varpi\sqrt{3}}{\pi^2}.$$

As the part of the earth concerned here is only a small part of the land surface, it was desirable in the computation to take four times as many points as were previously chosen. The intervals in longitude were 10° , in latitude 5° . From data based on the considerations given above I find

$$\iint \varpi \sin^2 \theta \cos \theta \cos \phi d\theta d\phi = 3.06; \quad \iint \varpi \sin^2 \theta \cos \theta \sin \phi d\theta d\phi = 1.63.$$

Accumulations of snow on the area considered must be associated with a corresponding diminution of mass over the rest of the earth. This is roughly allowed for by applying a correcting factor 1.2 to the values of F and G based on the above formulæ.

* Milham, *Meteorology*, chart xxvi.

These give in all then

$$\lambda_0 = -0''.0238 \cos \odot + 0''.0175 \sin \odot$$

$$\mu_0 = -0''.0126 \cos \odot + 0''.0093 \sin \odot.$$

VI. *The Effect of Vegetation.*

As has been already remarked, data with regard to the amount of rain (as distinguished from snow) that is retained in the form of soil-moisture seem to be completely lacking. In addition to this part, however, a portion of the rainfall becomes converted into plant tissue by vegetative processes. At the same time gaseous constituents are absorbed from the atmosphere. This abstraction from the air is included, as far as its influence on the atmosphere is concerned, in the variation of atmospheric pressure. The formation of plant tissue in this way must then be regarded as a cause by which matter tends to be accumulated over a particular area of the surface during summer, and dissipated in winter.

During the summer the vegetative parts of plants increase in mass in two ways. In trees, large quantities of sap rise from the ground, and thus the woody parts become heavier. Leaves are also produced in deciduous trees. In herbs, the whole of the subaerial portion is regenerated annually in the earlier part of the summer. Later in the year, usually in late summer or early autumn, the subaerial parts of all terrestrial plants partially dry up, and ultimately herbs wilt and fall to the ground, while deciduous trees cast all their leaves. The dead portions continue to lose weight until decomposition is complete. We may say then that there is a periodic part of the mass of trees, shrubs, and herbs which has a maximum in summer and a minimum in winter.

Precise quantitative estimates of this periodic variation seem difficult to obtain. From observations in a beechwood on the Gogmagog Hills, Cambridge, I estimate that in July the living leaves per square centimetre weigh 2.68 grams, while the previous year's dead leaves weigh 1.14 grams. In January the most recent dead leaves amount to about 1.5 grams per square centimetre. As the previous instalment of dead leaves must have lost weight by decomposition between July and the succeeding January, we see that the difference between July and January amounts to about 1.5 grams per square centimetre.

The estimation of sap is more difficult. According to Schlich* the amount of moisture in freshly felled wood is about 45 per cent., and his table of six trees shows a maximum in July for four of them, while the minima preponderate about April. There is thus evidence of a shorter period as well as the annual one. In forest land the weight of timber per square centimetre may reach 5 grams, so that the difference of the weights in July and January may be taken to be about 2 grams. This effect concerns both evergreen and deciduous trees.

* *Manual of Forestry*, vol. v. p. 13.

As an example of a fairly typical natural grassland, a patch of *Nardus stricta* on Birtley Fell, Durham, was examined. The loss of weight of the subaerial parts of this plant on drying was found to be about 1 gram per square centimetre. Here also the maximum probably comes in July, at the time of the greatest production of flowers.

In all, then, it seems likely that the order of magnitude of this periodic part is such that it has a range of about 2 grams per square centimetre in our latitude. This range is zero at the equator, where the vegetation seems to have no annual period. As the poles produce little effect on account of the small factor $\sin^2 \theta$ in $yzd\mathbf{S}$, we shall not be far wrong in assuming that m is proportional to $\cos \theta \cos (\odot - 114^\circ)$. Then over the land surface we have $\sigma = -1.3 \cos \theta$; $\sigma' = +3.0 \cos \theta$. As the area of the earth that is covered with land is $\frac{2}{7}$ of the whole surface, we must multiply by a factor $\frac{7}{5}$ in finding λ_0 and μ_0 , to allow for the fact that while the mass on land is increasing, that of the ocean is diminishing, and the two effects act in the same direction when we find the products of inertia.

We find then

$$\begin{aligned} \lambda_0 &= -1.4 \times 9''.1 \times 10^{-3} \int_0^\pi \int_0^{2\pi} (-1.3 \cos \odot + 3.0 \sin \odot) \sin^2 \theta \cos^2 \theta \\ &\hspace{25em} \cos \phi d\theta d\phi \\ &= 0''.0017 \cos \odot - 0''.0038 \sin \odot. \end{aligned}$$

$$\begin{aligned} \mu_0 &= -1.4 \times 9''.1 \times 10^{-3} \int_0^\pi \int_0^{2\pi} (-1.3 \cos \odot + 3.0 \sin \odot) \sin^2 \theta \cos^2 \theta \\ &\hspace{25em} \sin \phi d\theta d\phi \\ &= 0''.0024 \cos \odot - 0''.0055 \sin \odot. \end{aligned}$$

The effect of vegetation is therefore a small one.

VII. *The Effect of Polar Ice.*

Accumulation of ice and snow round the poles may be summed up under two types.

The first includes the deposition of snow during the winter, with its removal partly by evaporation and partly by glacier flow during the summer. This part would correctly be included under snowfall. On account, however, of the symmetry of the permanently frozen areas about the poles it seems very unlikely that this portion can contribute anything appreciable to the movement of the instantaneous axis.

The second type is due to the freezing of the ocean. Freezing in mid-ocean would necessarily lead to no change in the mass per unit area, since the ice must float in such a way that the weight of the ice is equal to that of the water displaced. Ice attached to the edge of a continent may behave differently near the margin, but even in this case, on account of symmetry, the contribution to the motion of the axis must be small.

VIII. *Summary.*

The contributions to the motion of the point ($\lambda_0\mu_0$) from the respective causes considered in this paper are given in the following table:—

	λ_0	μ_0
Atmospheric and oceanic motions	$0''.0203 \sin \odot + 0''.0128 \cos \odot$	$0''.0530 \sin \odot - 0''.0067 \cos \odot$
Snowfall	$0''.0175 \sin \odot - 0''.0238 \cos \odot$	$0''.0093 \sin \odot - 0''.0126 \cos \odot$
Vegetation	$-0''.0038 \sin \odot + 0''.0017 \cos \odot$	$-0''.0055 \sin \odot + 0''.0024 \cos \odot$
Total	$0''.0340 \sin \odot - 0''.0093 \cos \odot$	$0''.0568 \sin \odot - 0''.0169 \cos \odot$

The motion found is almost rectilinear, but direct. If we now put $\lambda = \kappa\lambda_0$, $\mu = \kappa\mu_0$, and make $\kappa = \frac{3}{4}$, we have

$$\begin{aligned}\lambda &= 0''.0255 \sin \odot - 0''.0070 \cos \odot \\ \mu &= 0''.0426 \sin \odot - 0''.0127 \cos \odot.\end{aligned}$$

On comparing these theoretical values with those deduced from observations, we see that for every term except that in λ involving $\sin \odot$ the values of the coefficients derived from known meteorological causes lie between the greatest and least of those derived from observations of the movement of the pole. The observational values for this exceptional term range from $-0''.068$ to $+0''.015$, so that it is just possible that it may lie within the limits of experimental error and irregular variation. If its coefficient were reduced to zero, this would be sufficient to make the whole motion retrograde. The theory gives a particularly good representation of the largest term in the motion, namely, that in μ that is proportional to $\sin \odot$.

The above determination of the movement of the pole of inertia gives for that of the pole of rotation the co-ordinates

$$\begin{aligned}x &= -0''.096 \sin \odot - 0''.103 \cos \odot \\ y &= +0''.080 \sin \odot - 0''.101 \cos \odot.\end{aligned}$$

This has to be compared with the values obtained by Kimura.

$$\begin{aligned}x &= 0''.011 \sin \odot - 0''.096 \cos \odot \\ y &= 0''.051 \sin \odot + 0''.004 \cos \odot\end{aligned} \left. \vphantom{\begin{aligned}x &= 0''.011 \sin \odot - 0''.096 \cos \odot \\ y &= 0''.051 \sin \odot + 0''.004 \cos \odot\end{aligned}} \right\} \text{from } 1893.8 \text{ to } 1899.8$$

$$\begin{aligned}x &= 0''.008 \sin \odot - 0''.064 \cos \odot \\ y &= 0''.056 \sin \odot - 0''.006 \cos \odot\end{aligned} \left. \vphantom{\begin{aligned}x &= 0''.008 \sin \odot - 0''.064 \cos \odot \\ y &= 0''.056 \sin \odot - 0''.006 \cos \odot\end{aligned}} \right\} \text{from } 1900.0 \text{ to } 1907.0$$

and with my results, given later in this paper,

$$\begin{aligned}x &= 0''.038 \sin \odot - 0''.055 \cos \odot \\ y &= 0''.061 \sin \odot + 0''.017 \cos \odot\end{aligned} \left. \vphantom{\begin{aligned}x &= 0''.038 \sin \odot - 0''.055 \cos \odot \\ y &= 0''.061 \sin \odot + 0''.017 \cos \odot\end{aligned}} \right\} \text{from } 1907.0 \text{ to } 1913.9.$$

While in its general features the theoretical motion resembles the actual motion, the terms in $\sin \odot$ in x and $\cos \odot$ in y are about three times as large as they should be, and the two other terms

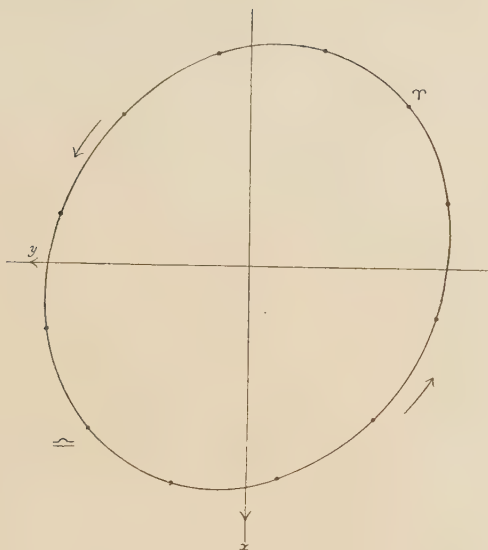
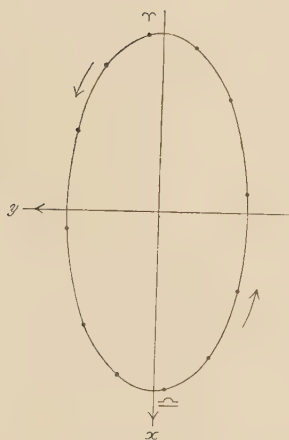
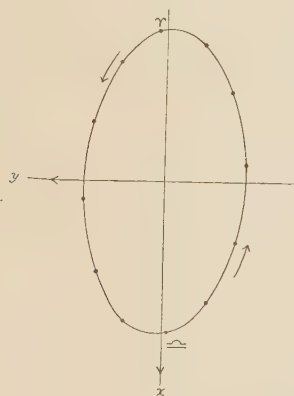
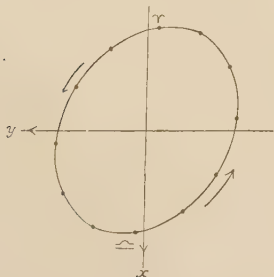
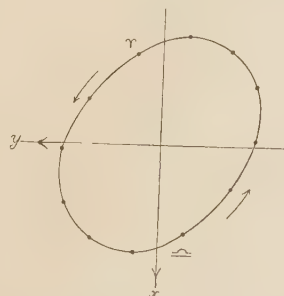


Fig. 1.—Theoretical Motion.

Fig. 2.—Observed Motion, 1893-1896
(Bakhuyzen).Fig. 3.—Observed Motion, 1893-8
to 1899-8.Fig. 4.—Observed Motion, 1900-0
to 1907-0.Fig. 5.—Observed Motion, 1907-0
to 1913-9.

are somewhat larger than the best observed values.* The theoretical orbit is thus about 40° wrong in phase. The former discrepancy arises directly from that already noted in the term in λ involving $\sin \odot$. As this term comes from the combination of two very badly determined parts, it is quite likely to be seriously in error.

The results are then as follows:—

(1) The known meteorological causes are apparently capable of giving a fairly good account of the observed annual motion of the pole, the errors found being perhaps within the range of uncertainty of the data.

(2) Attempts to explain the discrepancies that remain must be mainly directed to finding a cause that is capable of displacing the pole of inertia at the March equinox in the direction opposite to the Greenwich meridian.

(3) As the phenomenon is essentially complex in character, the existence of a large irregular portion in the movement is not surprising.

IX. Reduction of the Observations from 1907'0 to 1914'0.

In order to get a further test of the theory it has seemed desirable to reduce the more recent data, given by Albrecht in the *Astronomische Nachrichten*. To treat the annual and 14 monthly parts of the polar motion separately an interval of 7 years was chosen. The positions of the pole in this interval were as follows, the intervals between consecutive observations being a tenth of a year, and the unit one second of arc.

Values of x .

Date.	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9
1907	'044	'063	'067	'065	'037	'003	'043	'101	'147	'137
08	'081	'014	'118	'195	'235	'209	'134	'029	'114	'210
09	'257	'242	'162	'007	'189	'289	'318	'266	'109	'072
10	'184	'262	'296	'231	'073	'120	'268	'326	'299	'195
11	'019	'123	'200	'221	'160	'081	'098	'173	'250	'279
12	'209	'119	'016	'094	'131	'120	'091	'062	'008	'054
13	'099	'133	'151	'139	'110	'061	'002	'053	'089	'098

Values of y .

Date.	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.
1907	'122	'064	'013	'028	'063	'081	'063	'017	'045	'123
08	'193	'227	'204	'136	'041	'067	'141	'165	'152	'089
09	'029	'154	'268	'327	'303	'180	'026	'114	'207	'253
10	'234	'090	'078	'241	'327	'345	'266	'088	'080	'210
11	'271	'238	'119	'037	'209	'321	'353	'296	'172	'030
12	'078	'146	'157	'106	'026	'078	'159	'193	'210	'203
13	'157	'107	'052	'012	'056	'069	'066	'048	'008	'054

* See Plate 8.

If the period of the free vibration was exactly 14 months, it would be completely eliminated if we simply averaged the vertical columns of these tables. The actual period is such that six periods of the free motion is about 7 years 1 month, so that there will be no great error if we assume that the elimination can be performed similarly here.

The average co-ordinates for each tenth of a year are given by

	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.
$x_0 =$	-.022	-.043	-.029	-.020	+.019	.069	.097	.083	.043	-.001
$y_0 =$	-.012	+.011	.048	.085	.105	.101	.075	.033	-.003	-.020

The co-ordinate z is regarded as systematic experimental error and is not here considered.

Analysing these harmonically to find the annual and constant terms, we find that the best approximation is given by

$$x = -0''.0474 \sin \chi - 0''.0470 \cos \chi + 0''.0196$$

$$y = +0''.0270 \sin \chi - 0''.0576 \cos \chi + 0''.0423$$

where χ is the sun's longitude measured from the beginning of the year. If x_1 and y_1 be the values calculated from these formulæ, we have

	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.
$x_1 =$	-.027	-.046	-.040	-.010	+.030	.067	.086	.080	.050	.010
$y_1 =$	-.016	+.011	.050	.086	.105	.100	.073	.034	-.002	-.021
$x - x_1 =$	.005	.003	.011	-.010	-.011	+.002	.011	.003	-.007	-.011
$y - y_1 =$	.004	.000	-.002	-.001	.000	+.001	.002	-.001	-.001	+.001

The simple expressions x_1 and y_1 thus represent x and y to within about a tenth of their amount. The residuals indicate a semi-annual motion of small amplitude, but as its phase differs considerably from that indicated by Kimura, it would be dangerous to place much reliance on it.

Putting now $\chi = \odot + 80^\circ$, we find for the annual and constant parts

$$x_1 = 0''.0380 \sin \odot - 0''.0548 \cos \odot + 0''.0196$$

$$y_1 = 0''.0614 \sin \odot + 0''.0166 \cos \odot + 0''.0423.$$

These agree fairly well with the two Kimura determinations. The method is the same as Kimura's, so that the results are strictly comparable.

The outstanding parts of the polar motion are given in the following tables.

Date.	$x - x_0$									
	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.
1907	.066	.106	.096	.085	-.056	-.066	-.140	-.184	-.190	-.136
08	-.059	.057	.147	.215	.216	.140	.037	-.054	-.157	-.209
09	-.235	-.199	.133	.027	.170	.220	.221	.183	.066	-.071
10	-.162	-.219	-.267	-.211	-.092	.051	.171	.243	.256	.196
11	.041	.080	-.171	-.201	-.179	-.150	.001	.090	.207	.280
12	.231	.162	.045	-.074	-.150	-.189	-.188	-.145	-.051	.055
13	.121	.176	.180	.159	-.091	-.008	-.099	-.136	-.132	-.097

Date.	$y - y_0$									
	'0.	'1.	'2.	'3.	'4.	'5.	'6.	'7.	'8.	'9.
1907	'134	'053	- '035	- '113	- '168	- '198	- '138	- '050	'048	'143
08	'205	'216	'156	'051	- '064	- '168	- '216	- '198	- '149	- '069
09	'041	'145	'220	'242	'298	'079	- '049	- '147	- '204	- '233
10	- '222	- '101	'030	'156	'222	'244	'191	'055	- '077	- '190
11	- '259	- '249	- '167	- '048	'104	'220	'278	'263	'175	'050
12	- '090	- '157	- '205	- '191	- '131	- '023	'084	'160	'213	'223
13	'169	'096	'004	- '097	- '161	- '170	- '139	- '081	- '005	'074

These co-ordinates give an indication of a term with a period of about 15 months, as well as the 14-monthly term. The apparent amplitude of the 14-monthly motion shows a variation in consequence of this, but the series of observations is not long enough to make it possible to determine either the period or the amplitude of this term satisfactorily. The existence of a term whose period is 15.98 months has been indicated in the azimuth errors of the Cape Transit Circle by Professor Turner.* The effects of any variation in the amplitudes of the annual motion will also be included in the residuals of these tables.

The Kinetic Energy of a Star Cluster. By A. S. Eddington,
M.A., M.Sc., F.R.S., Plumian Professor.

1. *In any Star Cluster in a steady state the Internal Kinetic Energy is one-half the exhaustion of Potential Energy.*—Although I cannot find that this result has ever been explicitly stated, it follows almost immediately from formulæ found in the theory of gases in connection with the virial.† It will probably be more convenient to astronomers to give in full the proof for stars, instead of quoting these formulæ. It will be seen that the result does not in any way depend on the special conditions of a gas, but is a perfectly general consequence of the law of gravitation.

Taking the centre of gravity of the cluster as origin, let

- x, y, z be the co-ordinates of a star of mass m ,
- X, Y, Z the gravitational force on m , due to the attractions of the other stars of the cluster,
- T the total kinetic energy of the motions of the stars relative to the centre of gravity,
- Ω the exhaustion of potential energy of the cluster,
- C the moment of inertia of the cluster about its centre of gravity.

* Turner, *Tables for facilitating Harmonic Analysis*, p. 41.

† Cf. Jeans, *Dynamical Theory of Gases*, first edition, p. 145, where the virial is found to $-\frac{1}{2}\Sigma\Sigma r\phi(r)$. Put $\phi(r) = -1/r^2$, and the result follows immediately. I have since learnt that the result has been given by H. Poincaré (*Hypothèses cosmogoniques*, p. 94) in a review of Ligondés' hypothesis of a swarm of meteorites. I am indebted to Mr. H. Jeffreys for this reference.

We have

$$m \frac{d^2 x}{dt^2} = X.$$

From the identity

$$\frac{1}{2} \frac{d^2}{dt^2} (x^2) = \left(\frac{dx}{dt} \right)^2 + x \frac{d^2 x}{dt^2}$$

we have

$$\frac{1}{4} m \frac{d^2}{dt^2} (x^2) = \frac{1}{2} m \left(\frac{dx}{dt} \right)^2 + \frac{1}{2} x X.$$

Add this equation and the two corresponding equations for y and z , and sum for all the stars of the cluster; the result is

$$\frac{1}{4} \Sigma m \frac{d^2}{dt^2} (r^2) = \frac{1}{2} \Sigma m v^2 + \frac{1}{2} \Sigma (xX + yY + zZ) \quad . \quad . \quad (1)$$

where $r^2 = x^2 + y^2 + z^2$, and v is the velocity of the star.

$$\begin{aligned} \text{The left-hand side} &= \frac{1}{4} \frac{d^2}{dt^2} \Sigma m r^2 \\ &= \frac{1}{4} \frac{d^2 C}{dt^2} \end{aligned}$$

This evidently vanishes in a steady state, because the moment of inertia cannot be altering. Equation (1) therefore reduces to

$$T = -\frac{1}{2} \Sigma (xX + yY + zZ) \quad . \quad . \quad (2)$$

To evaluate (2), we remark that X is the sum of the attractions on a particular star of all the other stars, so that the whole expression may be split up into separate terms corresponding to every possible pair of stars. Consider a pair of stars m_1 and m_2 . The contribution due to the attraction of m_2 on m_1 is

$$-\frac{1}{2} x_1 \frac{m_1 m_2 (x_2 - x_1)}{r_{12}^3} - \frac{1}{2} y_1 \frac{m_1 m_2 (y_2 - y_1)}{r_{12}^3} - \frac{1}{2} z_1 \frac{m_1 m_2 (z_2 - z_1)}{r_{12}^3}$$

where

$$r_{12}^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2.$$

The portion due to the attraction of m_1 on m_2 is

$$-\frac{1}{2} x_2 \frac{m_1 m_2 (x_1 - x_2)}{r_{12}^3} - \frac{1}{2} y_2 \frac{m_1 m_2 (y_1 - y_2)}{r_{12}^3} - \frac{1}{2} z_2 \frac{m_1 m_2 (z_1 - z_2)}{r_{12}^3}.$$

Adding these together, they give

$$\frac{1}{2} \frac{m_1 m_2}{r_{12}}.$$

Hence

$$-\frac{1}{2} \Sigma (xX + yY + zZ) = \frac{1}{2} \Sigma \frac{m_1 m_2}{r_{12}},$$

the latter sum being taken over all combinations of stars in pairs.

$$= -\frac{1}{2} \text{ (potential energy of cluster).}$$

Hence (2) gives the result enunciated, namely,

$$T = \frac{1}{2}\Omega \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The whole energy of the cluster is $T - \Omega$, and the result may be expressed in the form—

Potential energy is twice the whole energy.

Kinetic energy is minus the whole energy.

2. It is of some interest to notice the general formula when the state is not steady. Equation (1) then gives

$$\frac{1}{4} \frac{d^2C}{dt^2} = T - \frac{1}{2}\Omega \quad . \quad . \quad . \quad . \quad . \quad (4)$$

so that the result holds for a roughly steady state, if d^2C/dt^2 is negligible.

Moreover, if $T > \frac{1}{2}\Omega$, d^2C/dt^2 is positive, so that the cluster expands (or its rate of contraction diminishes so that there is ultimately an expansion). Usually expansion diminishes Ω , and consequently diminishes $T - \frac{1}{2}\Omega$, the whole energy $T - \Omega$ remaining constant. Similarly, if $T < \frac{1}{2}\Omega$, there is a tendency for $T - \frac{1}{2}\Omega$ to increase. We see, therefore, that the value of $T - \frac{1}{2}\Omega$ is likely to be accelerated towards zero; but, since there is no absolute relation between C and Ω , we cannot draw any definite conclusion without further data as to the constitution of the cluster. Perhaps the cluster will attain the steady state by a series of rapidly damped oscillations of $T - \frac{1}{2}\Omega$.

3. *Rate of Dissolution of a Moving Cluster.*—This example is appended to illustrate the use that can be made of the foregoing result. It has been shown by Jeans* that the parallelism of the motions of stars in a cluster is destroyed with exceeding slowness by perturbations due to the passage of other stars through the cluster. But although the deflections of motion are very small, they will lead to a comparatively rapid scattering of the stars in space unless counteracted by some other force such as the mutual attraction of the cluster. For example, from the present dimensions of the Taurus cluster I have shown that it could not have existed more than 57 million years unless the scattering had been counteracted.† We are now in a position to take this mutual attraction into account and estimate the actual rate of scattering.

The chance perturbations by external stars produce a probable deflection of the motion of any cluster star proportional to the square-root of the time, and therefore give it a probable kinetic energy (relative to the centroid of the cluster) proportional to the time elapsed. This internal energy is acquired at the expense of the translational energy of the cluster as a whole or of the stars encountered. Let M be the mass of the whole cluster, a the velocity relative to the cluster acquired by a star in unit time (mean-square value); then the cluster will be gaining internal

* *Monthly Notices*, vol. lxxiv. p. 109.

† *Stellar Movements and the Structure of the Universe*, p. 254.

energy at the rate $\frac{1}{2}Ma^2$. By § 1 the rate of increase of potential energy of the cluster will then be Ma^2 , provided an approximately steady state is maintained. (Since the cluster stars are closer and their relative motions much slower than the external stars, their mutual encounters will be more potent as a means of transferring energy; that is to say, the energy will be redistributed within the cluster more rapidly than it is acquired from outside, and a steady state should result.) Now the potential energy of the cluster is $-M^2/2c$, where c is proportional to the linear dimensions of the cluster and is of the same order of magnitude as the average radius. For Plummer's law of density, $(1+r^2)^{-\frac{5}{2}}$, c is about 0.82 times the median radius of the cluster. Thus we have

$$\frac{d}{dt}\left(-\frac{M^2}{2c}\right) = Ma^2 \quad . \quad . \quad . \quad . \quad (5)$$

or, integrating for an interval t ,

$$\frac{1}{c_0} - \frac{1}{c} = \frac{2a^2t}{M}.$$

The time taken for the cluster to increase from half its present linear dimensions to its present size is thus

$$\tau = \frac{M}{2ca^2} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Taking Jeans's figures for the Taurus cluster, he finds an average deflection of $1'$ in a million years, which, for a velocity of the cluster of 40 km./sec., is equivalent to a transverse velocity of 0.012 parsecs per million years. If we take for the unit of mass the sun's mass, and for the unit of time a million years, the unit of length must be 0.165 parsecs if the gravitational constant is eliminated. Hence $a = 0.073$. We may take c as roughly 3 parsecs, or 18 units. Then (6) gives

$$\tau = M \times 5 \text{ million years.}$$

Forty stars are known to belong the cluster, and there are doubtless many others. Thus τ must be several hundred million years at least, and the present concentration of the cluster does not indicate an unduly short period of existence.

A Determination of the Mean Parallax of Stars for different Magnitudes. By H. W. Unthank, B.A. (With an Introduction by A. S. Eddington, M.A., F.R.S.)

Introduction (by Professor A. S. Eddington).

The determination of the rate at which the mean distance of the stars increases with progressive decrease of brightness is one of the fundamental problems of stellar statistics. Hitherto the investigation made by J. C. Kapteyn in 1901 has been generally used. For practical purposes, such as obtaining the mean parallax of the comparison stars in parallax investigations, his results are probably sufficient; but in connection with the investigation of the general distribution of the stars in space there have been suggestions that his formula gives a too rapid increase of distance with magnitude. In the fifteen years that have elapsed many more proper motions have become available, and their systematic errors have perhaps become better known, so that a new research on this subject seemed desirable.

A related problem discussed by Kapteyn, viz. the mean parallax for given magnitude *and* proper motion, is not dealt with in this paper. It has recently been discussed in a memoir by P. J. van Rhijn. The two problems are quite distinct, and it does not seem possible to derive the solution of one from the other.

The actual measures of parallax can throw little light on the present problem, and the work must depend entirely on proper motions. In particular it is necessary to use a catalogue of proper motions covering a considerable range of magnitude. The straightforward method is to group the stars according to magnitude and calculate the mean parallactic motion of each group; this can at once be translated into mean parallax, knowing the solar motion in linear measure. The great drawback is that some of the stars have excessively great motions, calling for somewhat arbitrary exclusion; and even after excluding these, the mean will depend mainly on the moderately near stars; a large number of more distant stars might be removed to infinity with very little change in the result. Regarded from another point of view, the mean parallax in its strict sense is not a good measure of the distribution of the stars in distance; it would be better to use the mean distance, or, failing that, some other measure of the distribution which gives reasonable weight to the more distant stars.

The method which I proposed to Mr. Unthank is an attempt to avoid this difficulty; but naturally in abandoning the direct method it was necessary to introduce certain assumptions which may appear objectionable. The method is fully described in the paper, and I will here only mention one point in its favour, to which I attach some importance. For convenience it is stated as

an assumption that the stars of given magnitude are arranged at distances according to the law $re^{-h^2k^2r^2}$, and the coefficient hk is taken as proportional to the mean parallax. But the law is not used so much for an *assumption* as for a *standard* of comparison. In taking the arithmetic mean distance we find the single distance which most nearly fits all the stars, judged by the usual criterion that the sum of the squares of the residuals is a minimum; but it would be absurd to suggest that in using the arithmetic mean we are assuming that all the stars are at this single distance. So here the constant hk is determined so that the curve $re^{-h^2k^2r^2}$ most nearly fits the distribution of the stars, judged by the least-squares criterion; and, when this is done, hk clearly gives a measure of the distance of the group as compared with a similar group of different brightness. But it is not necessarily assumed that the stars actually follow the curve, though the method would lose appropriateness if the distribution deviated too widely. The deviation would have to be very wide indeed if this comparison distribution were not more suitable than that implied in taking the arithmetic mean.

The other assumption is certainly open to criticism. The distribution of the linear individual velocities of stars was taken to be according to the law $e^{-h^2v^2}$. It seems probable now that h depends on the absolute magnitude of the stars, but this was not known when the investigation was begun. I do not think, however, that this is likely to have any systematic influence on the results based on the hk table.

In the second determination, based on the values of k , h is assumed not to vary with magnitude. But it is clear that if h depends on the absolute magnitude it will also be correlated with the apparent magnitude. The modern evidence seems to show that h diminishes for the fainter stars, and therefore the results of the second determination will give too small a diminution of parallax with decreasing brightness. As is pointed out in the paper, the second determination is in other respects—smaller probable error and independence of systematic error—more satisfactory than the first, so that this theoretical uncertainty is rather unfortunate. It may be hoped that before long we shall have data for estimating the possible correction that is required.

The absolute measures of distance—whether expressed by the constant hk or the mean parallax—are of little significance; the important point to determine is the factor by which the mean parallax (or mean distance) changes for a step of one magnitude. This was given by Professor Kapteyn as 0.78, and Mr. Unthank's result is 0.845. The change is in the direction demanded by recent criticism, *i.e.* that the mean parallax alters with magnitude less than had been supposed. The correction is not so great as I had anticipated; and, moreover, the possible correction referred to in the last paragraph would bring Unthank's result a little nearer to Kapteyn's.

It may be of interest to express the conclusions in another way,

taking two average stars which differ by $1^m.0$. Then, according to Kapteyn,

$0^m.54$ is accounted for by difference of distance, and hence
 $0^m.46$ is attributable to difference of absolute brightness.

According to Unthank,

$0^m.36$ is attributable to difference of distance, and
 $0^m.64$ is attributable to difference of absolute brightness.

The internal accordance of the results given in this paper is scarcely so good as might have been hoped; but it must be remembered that each separate determination of k and h rests on very few stars, and the final probable error is not unduly large. It would be unwise to base very much on these results, but I think this investigation is entitled to fair weight in conjunction with other researches. It seems clear that many more proper motions of faint stars will be needed before the problem can be solved with certainty.

§ 1. The following is a brief account of a problem I have been investigating under the direction of Professor Eddington, who suggested the method and has guided the work throughout. It is attempted to determine, by means of a least-squares method applied to proper motions, the mean parallax of different groups of stars, each group consisting of stars in a small region of the sky between certain limits of magnitude. The results are used to determine values of the constants in Kapteyn's formula $\log \varpi_m = \log a + m \log b$. The proper motions used are those given in Dyson and Thackeray's "New Reduction of Groombridge's Catalogue of Circumpolar Stars," and Boss's "Preliminary General Catalogue."

§ 2. Assumptions.

(1) If N = number of stars in a small region of the sky between fairly narrow limits of magnitude (*e.g.* 6.0 to 7.0), then the number lying at a distance between r and $r+dr$ is $2Nk^2h^2re^{-h^2k^2r^2}dr$, hk being independent of r . This formula has been shown by Sir F. W. Dyson and Professor Eddington independently* to represent fairly well the distribution of stars brighter than a limiting magnitude. It seems not unreasonable to suppose that it gives a fair approximation to the distribution of stars between limits of magnitude, especially as of the stars brighter than magnitude m , where m does not exceed 9, more than $\frac{2}{3}$ are fainter than magnitude $m-1$. (All the stars here considered have magnitudes brighter than 8.5 .)

(2) The first assumption at once gives the result that of the

* *M.N., R.A.S.*, lxxiii., No. 5 (both papers).

group of N stars the number at distances exceeding r is $N e^{-h^2/2r^2}$. It is now assumed that of these stars the proportion $\frac{h}{\sqrt{\pi}} e^{-h^2(v-V)^2} dv$ have component linear velocity between r and $v+dv$ measured relative to the sun, perpendicular to the line of sight, and along the great circle towards the sun's antapex, V being the component velocity of the sun in the same direction, and h being independent of r and r but depending on the region of the sky in which the stars lie. This assumption does not agree accurately with the Stardrift Theories, but is consistent with the Ellipsoidal Hypothesis. It can only be regarded as approximate, however, as there is reason to suppose that the mean linear velocity (or h) depends not only on the spectral type but also on the absolute luminosity.

§ 3. Theory.

From the assumptions it follows at once that the number of stars at distance $>r$ with component linear velocity between v and $v+dv$ is

$$\frac{Nh}{\sqrt{\pi}} e^{-h^2/2r^2} e^{-h^2(v-V)^2} dv.$$

This is the number of stars with linear velocity between v and $v+dv$ and resolved p.m. μ between 0 and v/r .

Let $v/r = \mu_1$. Then number of stars with linear velocity between v and $v+dv$ for which μ lies between 0 and μ_1

$$= \frac{Nh}{\sqrt{\pi}} e^{-h^2 \{k^2 \mu_1^2 + (v-V)^2\}} dv.$$

Now r is always positive, and v and μ_1 have the same sign. Hence, integrating with respect to v from 0 to $+\infty$, μ_1 remaining constant and positive, we have (after a little reduction):—number of stars for which $0 < \mu < \mu_1$

$$= \frac{N\mu_1}{\sqrt{\pi}\sqrt{(k^2 + \mu_1^2)}} e^{-k^2 + \mu_1^2} \int_{-\frac{Vh\mu_1}{\sqrt{(k^2 + \mu_1^2)}}}^{+\infty} e^{-x^2} dx,$$

or

$$\frac{N}{2Vh} e^{-V^2h^2} Q \left\{ \frac{Vh\mu_1}{\sqrt{(k^2 + \mu_1^2)}} \right\},$$

where

$$Q(z) = \frac{2}{\sqrt{\pi}} z e^{z^2} \int_{-z}^{\infty} e^{-x^2} dx.$$

The number of stars for which $\mu_2 < \mu < 0$, i.e. $0 < -\mu < -\mu_2$, is found by taking the opposite direction as positive and writing $-V$ for V and $-\mu_2$ for μ_1 . The result is

$$-\frac{N}{2Vh} e^{-V^2h^2} Q \left\{ \frac{Vh\mu_2}{\sqrt{(k^2 + \mu_2^2)}} \right\}.$$

$Q(z)$ is seen to be continuous and $Q(0) = 0$.

Write $z_1 = Vh\mu_1/\sqrt{(k^2 + \mu_1^2)}$ and $z_2 = Vh\mu_2/\sqrt{(k^2 + \mu_2^2)}$.

Then, whatever the signs of μ_1 and μ_2 , the number of stars for which $\mu_1 > \mu > \mu_2$

$$= \frac{N}{2Vh} e^{-V^2 h^2} [Q(z_1) - Q(z_2)] \quad . \quad . \quad . \quad (A)$$

An approximate value of Vh may be found at once from the proper motions and the equation

(Number of positive p.ms.) - (number of negative p.ms.)

$$= N[Q(Vh) + Q(-Vh)] / (2Vhe^{V^2 h^2}) = \frac{2N}{\sqrt{\pi}} \int_0^{Vh} e^{-x^2} dx.$$

Again, from the p.ms. and this approximate value of Vh an approximate value of k can be obtained.

Let Vh_0 , k_0 be the approximate values found, and let $h = h_0 + \Delta h$, $k = k_0 + \Delta k$.

Denoting $\partial Q(z)/\partial h$ by $Q'(z)$ and the left-hand side of equation (A) by N' , and making use of the first differentials of the right-hand side with respect to h and k , we obtain after some reduction:—

$$\begin{aligned} & -k_0 \Delta k [z_1/(k_0^2 + \mu_1^2) Q'(z_1) - z_2/(k_0^2 + \mu_2^2) Q'(z_2)] \\ & + \frac{\Delta h}{h_0} [\{z_1 Q'(z_1) - z_2 Q'(z_2)\} - (1 + 2V^2 h_0^2) \{Q(z_1) - Q(z_2)\}] \\ & = 2Vh_0 e^{V^2 h_0^2} N' / N - \{Q(z_1) - Q(z_2)\} \quad . \quad . \quad . \quad (B) \end{aligned}$$

In this equation $z_1 = Vh_0 \mu_1 / \sqrt{(k_0^2 + \mu_1^2)}$ and $z_2 = Vh_0 \mu_2 / \sqrt{(k_0^2 + \mu_2^2)}$.

The formula (B) is an equation of condition for determining Δk and Δh .

§ 4. General Procedure.

The classification according to magnitude is as follows:—

(a)	brighter than	5.0		
(b)	„	„	6.0, not brighter than	5.0
(c)	„	„	7.0, „	„ 6.0
(d)	„	„	7.5, „	„ 7.0
(e)	„	„	8.0, „	„ 7.5
(f)	„	„	8.5, „	„ 8.0

For all the classes except (a) Groombridge stars were used. For (a) and (b) Boss stars were used, so that two solutions were found for the fifth magnitude stars.

The five regions of the sky considered are bounded by meridians of R.A. 4^h apart and by parallels of Dec. 27° apart. The limits of Dec. are in each case +38' to +65°. Those of R.A. are 20^h, 24^h, 4^h, 8^h, 12^h, and 16^h. The region between the meridians 16^h and 20^h was not treated, because of the proximity of the solar apex to its boundary. It was also less convenient to treat the region surrounding the north pole, and this too was neglected.

For each star the resolved component of p.m. in seconds of arc along the great circle towards the solar antapex was found.

To minimise the labour the position angle of this point, assumed to be at R.A. 93° Dec. $-29\frac{1}{2}^\circ$, was found only for the centre of each region, and this value used for all the stars in the region. V , the corresponding component of the sun's linear velocity, was also calculated for the centre of each region, the sun's velocity being taken as $19\cdot5$ km./sec.

In the case of the Groombridge stars a correction, computed from tables at the end of Boss's Catalogue, was applied for systematic errors of the proper motions.

The resolved proper motions being classified according to region and magnitude, approximate values of Vh and k were calculated or guessed for each group. Equations of condition of the form (13) were then formed, corresponding to successive ranges μ_1, μ_2 .

§ 5. As it was supposed that the inaccuracies in the equations of condition consisted chiefly of errors of sampling, the way to give the equations equal weight appeared to be to make each refer to an equal number of stars, *i.e.* to let the quantities N' be all equal (for the same group of stars), except in the case of the two equations for which one limit of proper motion is infinite. In these two equations, the chances of error being reduced, N' was chosen to have rather more than half its usual value. The smallest value of N' used was about 4, and subject to this condition the number of equations formed was generally 8 to 10 for a group of stars.

It may be proved without difficulty from the assumptions that provided Vh does not exceed $\cdot685$ the frequency distribution of resolved p.m.s. has only one maximum. (In only 3 out of the 35 groups was Vh found to exceed this quantity $\cdot685$, and it is extremely likely that in these cases also there should be only one maximum.) In choosing the values of the various limits μ_1 and μ_2 , therefore, an attempt was made to ensure that the sequence of quantities $N' / (\mu_1 - \mu_2)$ should have only one maximum. In many cases this was impossible; this phenomenon has, of course, been observed before, and forms part of the evidence in favour of the Drift Theories as opposed to the Ellipsoidal.

It was decided that no great improvement would be effected by correcting the frequency distribution for the probable accidental errors of the proper motions.

The normal equations when solved gave values of $\Delta h/h_0$ and $\Delta(k^2)$. In most cases these corrections were not small, and the corrected values of Vh and k were used as initial values Vh_0, k_0 for a second approximation. Often the first values of Δh and Δk obtained were not very close to the true corrections, but further approximations appeared to lead eventually to the true values. In all cases approximations were persisted in until $\Delta h/h_0$ and $\Delta(k^2)/k^2$ were each less than $\frac{1}{10}$ in absolute value, and generally the final values were much above this standard of accuracy.

§ 6. The proper motions were expressed in units of $0''\cdot001$ per

year. The unit of distance being the parsec, it follows that the unit of linear velocity is 0.001 astronomical units per year. V , the resolved velocity of the sun, must be expressed in these units, and it has then the values shown below.

Region.	Limits of R.A.		$V \times 10^{-3}$.
	h	h	
α	20	to 24	3.02
β	0	,, 4	4.10
γ	4	,, 8	4.06
δ	8	,, 12	4.12
ϵ	12	,, 16	3.20

V , Vh , and k being now known, hk is found for each group.

Write λ for hk ; then, according to assumption (1), the number of stars in any one group at distances between r and $r+dr$ is $2N\lambda^2 r e^{\lambda^2 r^2} dr$.

Hence the (arithmetical) mean parallax = $2\lambda^2 \int_0^\infty e^{-\lambda^2 r^2} dr = \sqrt{\pi} \lambda$

or $\sqrt{\pi} hk$.

This result is expressed in seconds of arc, since r is expressed in parsecs.

The (arithmetical) mean distance = $2\lambda^2 \int_0^\infty r^2 e^{-\lambda^2 r^2} dr = \frac{\sqrt{\pi}}{2\lambda}$ parsecs.

The following tables show the values found for the quantities h , k , and hk :—

	$h \times 10^4$.							
Region.	Boss Stars.		Groombridge Stars.					Weighted Means (Groombridge only).
	<i>a.</i>	<i>b.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>	
α	1.18 ₄₁	1.30 ₉₂	1.69 ₉₉	1.91 ₂₃₇	1.62 ₁₈₇	1.29 ₁₅₃	1.05 ₁₉₂	1.523
β	1.41 ₃₇	.99 ₈₀	.81 ₈₀	1.31 ₁₄₀	1.55 ₉₀	1.75 ₇₃	1.72 ₁₀₀	1.424
γ	1.51 ₂₃	1.59 ₅₉	1.34 ₆₄	1.46 ₁₅₇	1.35 ₇₇	1.02 ₆₆	1.20 ₆₈	1.311
δ	1.37 ₂₅	1.00 ₃₇	.88 ₅₀	1.30 ₁₀₀	1.05 ₅₉	.42 ₅₇	1.87 ₄₀	1.097
ϵ	1.57 ₁₅	.94 ₄₄	1.03 ₅₈	.75 ₁₁₁	.54 ₅₃	.44 ₃₀	.44 ₃₆	.697

The suffixes are the numbers of stars used in each determination. They are used as weights. For all the Groombridge stars the weighted mean of h is 1.309×10^{-4} .

Region.	k .						
	Boss Stars.		Groombridge Stars.				
	a.	b.	b.	c.	d.	e.	f.
α	22.3	28.6	38.5	25.9	26.9	32.8	28.4
β	42.5	40.5	37.8	38.0	27.0	29.5	25.3
γ	44.8	47.9	48.3	37.3	34.4	26.9	28.4
δ	102.2	66.4	64.8	55.3	48.4	42.3	32.9
ϵ	166.9	69.1	60.3	43.2	46.2	26.0	41.8
Weighted means } f	...	45.5	47.5	37.1	33.1	32.0	29.8

Region.	Boss Stars.		$hk \times 10^3$.				
	<i>a.</i>	<i>b.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>
α	2'63	3'72	6'51	4'95	4'36	4'23	2'98
β	5'99	4'00	3'07	5'00	4'18	5'16	4'35
γ	6'77	7'62	6'48	5'45	4'64	2'74	3'41
δ	14'00	6'63	5'70	7'19	5'10	1'76	6'15
ϵ	26'2	6'50	6'22	3'23	2'49	1'16	1'85
Weighted means	{ ... 5'26		5'56	5'11	4'25	3'54	3'56

The weighted means of h show that this quantity varies systematically from region to region. It is to be expected, of course, that it should depend on the position relative to the velocity ellipsoid and on the angle between the directions to the solar apex and the vertex of star streaming. The variation cannot be explained completely this way, however, since regions α and ϵ give widely different values of h though they are very nearly symmetrical with respect both to the vertex and the apex. (The centres of these regions have Dec. 50° and R.A. 330° and 210° respectively.) It is more likely that the phenomenon is more closely associated with the variation in galactic latitude, on which the mean spectral type and mean luminosity depend.

It is at once seen from the tables that while the values of h are very irregular, both k and hk show in the means a definite though not very uniform progression with magnitude. This suggests that k may indicate more accurately than hk the variation of mean parallax with magnitude. The probable errors of $\log k$ are found to be generally smaller than those of $\log h$ (the average ratio appears to be about $\cdot 7$), and trial has shown that a systematic error in the proper motions which gives rise to an error of 30 per cent. in h may only lead to an error of 3 per cent. in k . It does not seem probable that such a systematic error can be present and vary widely and without regularity from magnitude to magnitude, but it was considered worth while to obtain a formula for k as well as for hk .

The regularity of the quantities in the tables was not good enough for the determination of a separate formula for each region, and only the weighted means of k and hk were used. The formula is thus intended to express the arithmetical mean parallax of the stars in the five regions. The brightest stars (a) were not used in the final calculation, but the result given by the method for these stars is compared with that found from the formula obtained.

It was necessary to correct the limits of magnitude given for the Boss stars. The Groombridge magnitudes are those of the Harvard Durchmusterung, and have been assumed to be on the scale of the Harvard Standard Sequence. The magnitudes represented by 4'9 and 5'8 on this scale are those which are represented by 5'0 and 6'0 respectively on Boss's scale.*

* Corrections given in Boss's Catalogue, p. xx.

§ 7. So far the mean parallaxes have been found for stars belonging to a finite range of magnitude. Thus the mean parallax of stars brighter than m_1 , but not brighter than m_2 , is known to be $p(m_1, m_2)$. Now, let m approximately = the mean magnitude of the stars in the range considered. By equating $\varpi(m)$ to $p(m_1, m_2)$ and assuming $\log \varpi(m) = \log a + m \log b$ we get a number of equations of condition which give approximate values of a and b . From these approximate values, with the help of the assumption that the logarithm of the number of stars between narrow limits of magnitude is expressible in the form $\beta + am$,* values of $\varpi(m)$ and of $\varpi(m_1, m_2)$, i.e. the mean parallax of the stars in the finite range, are calculated. The difference between these quantities is used as a correction to be applied to $p(m_1, m_2)$ to give a value for $\varpi(m)$, which is used to form a corrected set of equations of condition, leading to the final solution for a and b .

It is probable that the single result for the Boss stars is the most trustworthy of the six. Nevertheless, it was thought better that the equation should be treated as though of equal weight. The procedure in the determination of the formulæ for k and hk was exactly the same; the final formula in the former case was multiplied by the weighted mean value of h , in order to give values comparable with those given by the other formula. The probable errors of the quantities determined were found from the equations of condition. For convenience, Kapteyn's formula was used in the form

$$\log_{10} \varpi_m = \log_{10} \varpi_7 + v(m - 7).$$

§ 8. Results.

In formula I., obtained from the hk table, $\log_{10} \varpi_7 = 3.885 \pm .0090$ and $v = -.0725 \pm .0084$.

In formula II., obtained from the k table and the mean h , $\log_{10} \varpi_7 = 3.914$ and $v = -.0720 \pm .0052$.

$$\begin{array}{ll} \text{Or, I.} & \varpi_7 = 0''.00767 \quad \varpi_{m+1}/\varpi_m = .846 \pm .016. \\ \text{II.} & \varpi_7 = 0''.00820 \quad \varpi_{m+1}/\varpi_m = .847 \pm .010. \end{array}$$

The formula found by Kapteyn, when corrected by replacing the value 16.7 km. per sec. of the solar velocity by the modern value 19.5, gives

$$\varpi_7 = 0''.0093 \quad \varpi_{m+1}/\varpi_m = .78.^\dagger$$

As Kapteyn's magnitude factor .78 was generally regarded as rather low, it is satisfactory to obtain a larger value. The formulæ give smaller parallaxes for the stars of the magnitudes considered, and the proportionate difference is greater for the brighter stars. Presumably formula I. gives a better value of ϖ_7 ; the magnitude factor is sensibly the same in each formula, but has a smaller probable error in II.

* The values of a are taken from the results of the star counts by Chapman and Melotte as given in Table 31 on p. 189 of *Stellar Movements*.

† *Stellar Movements*, p. 205.

If we make the very doubtful assumption that for the stars concerned $a = .70$, *i.e.* that the logarithm of the number of stars within a certain range of magnitude is increased by $.70$ when the limiting magnitudes are each increased by unity, it can be shown that for stars brighter than magnitude 4.9 formula I. gives $0''.0105$ and formula II. $0''.0112$ for the mean parallax. The value found directly from the Boss stars within this range is $0''.0154$. It appears that changes in a do not greatly affect the above results, and therefore that the formulæ are not very suitable for the brightest stars.

With a view to determining the effect of galactic latitude, the means for each region of h , k , and hk were plotted against the cosines of twice the galactic latitude. No useful information appeared to be obtainable, however, perhaps largely because the points given by the first three regions were close together. Their position bore no obvious simple relation to those of the other two points.

Tenth Note on the Number of Faint Stars with Large Proper Motions. Zones $+25^\circ$, $+27^\circ$, and $+29^\circ$. By F. A. Bellamy, M.A., and J. D. McNeile, M.A.

References to previous notes of this series are:—*M.N.*, lxi. pp. 57 and 491; lxxi. pp. 45 and 582; lxxii. p. 65; lxxiv. pp. 26 and 27; lxxvi. pp. 102 and 428.

The present paper deals with the results derived from comparisons of ten pairs of plates referred to in Table I.

TABLE I.

Zones $+25^\circ$, $+27^\circ$, and $+29^\circ$. Stars with large P.Ms.

Plate Centre.		Total Stars compared.	Interval in Years.	Centennial P.Ms.			Limit of Centennial P.Ms. included.
Dec.	R.A.			All.	$15''$ to $20''$.	$< 70''$.	
$^{\circ}$	$^{\circ}$ $'$ $''$						$''$
$+25$	5 32	68	14.05	1	0	0	10.7
$+27$	5 32	385	10.03	5	3	2	15.0
$+29$	14 28	114	10.14	0	0	0	15.0
$+25$	19 0	293	10.02	1	1	0	15.0
$+27$	20 20	361	14.98	7	4	0	10.0
$+27$	21 0	313	9.92	4	2	2	15.1
$+25$	21 24	330	11.03	4	1	2	13.6
$+27$	21 32	538	10.00	7	4	3	15.0
$+29$	22 25	346	17.22	8	2	1	8.6
$+25$	22 44	437	11.81	14	7	2	12.7
Totals		3185	11.92 (mean)	51	24	12	13.0 (mean)

Only 13 of these 51 stars are of sufficient brightness to be included in the A.G. or other meridian Catalogues, the rest of the stars extending to magnitude 12.5. For the 3185 stars compared the limit of magnitude may be taken generally to include all stars as faint as 11.0 magnitude, though in some pairs of plates stars as faint as 13.0 are included.

The numbers of Proper Motions given for these ten plates are in each case slightly above the average number per square degree, as derived from the previous Notes. It may be remarked that, with the exception of plate +29°, 14^h 28^m, all the plates concerned in this paper are within, or closely contiguous to, the Milky Way.

The limit adopted for the inclusion of Proper Motions was $(\Delta x)^2 + (\Delta y)^2 = 25$ (in units of .001 interval or 0".3); the varying interval in years for the pairs of plates gives a varying limit to the centennial Proper Motion included, which is given in the last column of Table I.

Table II. contains the details of these 51 Proper Motions. The first column gives the R.A. of the plate centre (1900.0); the declination appears in the second column, prefixed to the number in the Oxford Astrographic Catalogue; the ϕ or photographic magnitude is in the third column, and is derived from the measured d , by use of the formulæ given in the Oxford volumes.

Column four has the number of the star in the Cambridge A.G.C., or in that of Berlin B (in these cases the letter B precedes the number). The centennial Proper Motions deduced from the comparisons are in the fifth column; the sixth and seventh give the differences Δx and Δy between the plates (unit of .0001 réseau interval or 0".03); and the last column contains the interval in years.

TABLE II.

List of Proper Motions in Zones +25°, +27°, and +29°.

R.A. of Plate Centre.	Oxford No. in Zone.	ϕ Mag.	A.G.C. No.	Cent. P.M. in Arc.	Δx .	Δy .	Interval in Years.
h m					Δx . Δy . (Unit .0001.)		
21 32	+27°63364	10.3		45.4	+150	+20	10.00
5 32	+27°12005	10.9		42.3	+100	-100	10.03
21 32	+27°63114*	7.4	12786	33.1	+110	-10	10.00
22 44	+25°79620*	4.6	B 8764	29.9	+115	-25	11.81
21 32	+27°63327	12.1		29.5	+90	-40	10.00
22 25	+29°65116	10.6		26.4	-41	-146	17.22
21 24	+25°73560*	8.3	12550	25.8	-59	-73	11.03
21 0	+27°61040	11.3		25.7	-30	-80	9.92
5 32	+27°11729	12.2		22.8	+70	+30	10.32
22 24	+25°79653	10.7		22.1	+86	+13	11.81
21 0	+27°61119*	8.1	12099	21.7	-60	-40	9.92
21 24	+25°73426	11.3		21.3	+70	-35	11.03

TABLE II.—*continued.*

R.A. of Plate Centre. h in	Oxford No. in Zone.	ϕ Mag.	A.G.C. No.	Cent P.M. in Arc.	$\Delta x.$ (Unit '0001.)	$\Delta y.$	Interval in Years.
22 44	+25°79852	11'3		19'7	- 77	+ 9	11'81
21 24	+25°73507	10'2		19'0	+ 61	+ 34	11'03
5 32	+27°11976	11'8		18'9	+ 20	- 60	10'32
22 25	+29°65326	10'5		18'7	- 40	- 100	17'22
20 20	+27°56887	10'9		18'4	- 70	- 60	14'98
20 20	+27°57292*	7'9	11179	18'4	- 60	- 70	14'98
21 32	+27°63006	11'1		18'2	- 10	- 60	10'00
21 32	+27°63479	12'3		18'2	+ 10	+ 60	10'00
22 44	+25°79765	11'1		17'7	- 25	+ 65	11'81
21 0	+27°61014	9'3		17'5	- 30	- 50	9'92
19 0	+25°59121*	6'1	B 6846	17'1	+ 39	+ 41	10'02
21 32	+27°63228*	8'9	12695	17'0	+ 40	+ 40	10'00
22 44	+25°79737	9'9		17'0	+ 52	+ 42	11'81
20 20	+27°56994*	10'1	11152	16'5	+ 80	+ 20	14'98
22 44	+25°79774	11'3		16'4	+ 35	+ 54	11'81
21 0	+27°61205	9'6		16'3	- 50	- 20	9'92
22 44	+25°79717	11'5		16'3	+ 12	+ 63	11'81
20 20	+27°57146	12'5		16'1	- 40	- 70	14'98
22 44	+25°79629	11'0		15'7	+ 36	+ 50	11'81
22 25	+29°65205*	10'2	13516	15'4	- 27	- 83	17'22
22 44	+25°79793	10'8		15'3	+ 53	- 16	11'81
21 32	+27°63535*	9'6	12669	15'0	+ 50	0	10'00
5 32	+27°11345	12'4		15'0	- 30	- 40	10'32
5 32	+27°11438	12'4		15'0	- 40	- 30	10'32
20 20	+27°57166*	8'4	11263	14'6	- 70	- 20	14'98
22 44	+25°79622	10'5		14'2	+ 56	0	11'81
21 24	+25°73514*	8'0	B 8213	14'2	+ 50	- 15	11'03
22 44	+25°79791	11'3		13'9	- 17	- 52	11'81
22 44	+25°79952	11'0		13'9	+ 54	- 8	11'81
22 44	+25°79673	10'7		13'0	+ 49	+ 14	11'81
22 44	+25°79610*	9'4	B 8749	12'1	+ 46	+ 13	11'81
5 32	+25°15505	9'9		12'0	+ 20	+ 50	14'05
22 25	+29°65435	10'3		11'0	- 58	- 26	17'22
22 25	+29°65104	11'3		10'7	- 37	- 49	17'22
20 20	+27°57352	11'7		10'2	- 10	- 50	14'98
20 20	+27°56878	9'1		10'0	- 40	- 30	14'98
22 25	+29°65403*	9'1	13508	9'2	- 17	- 50	17'22
22 25	+29°65250	10'6		9'2	+ 38	+ 37	17'22
22 25	+29°65271	10'5		8'6	- 15	- 47	17'22
22 25	+29°65138	10'5		7'2	- 22	- 35	17'22

The following notes may be of interest:—

Star +27° 63364 = +28° 65996 in L.P.M., Note 1, where the C.P.M. is 42".2.

Star +25° 73514 (B 8213) is given in L.P.M. 7 as 14".0 for the C.P.M.

Star +27° 61119 (C 12099) is also in L.P.M. 7 as 26".5 C.P.M., and it is in List B in the Oxford Astrographic Catalogue, viii. p. 92.

Star +27° 63114 (C 12786) is in List D, vol. viii. p. 121, where the P.Ms. are given from Cinc. xiii. 1748; these are equivalent to C.P.M. of 36".5.

The following stars are also in the same List B: Cambridge A.G.C., No. 11152, 11179, 12669.

Star +25° 79610 is μ Pegasi, which appears in Berlin B as a fundamental star (No. 8749).

Double Star, +29° 65138.

The following remarks are by F. A. Bellamy.

Though below the adopted limit, the fifty-second or last star in Table II. is added because, in the examination of the original measures with plate 536, it was found to have a faint star of the 11th magnitude close to it, and south following. The slightly fainter star, which only has one measurable image on the plate 536, is not given in the printed volume, and may for convenience of reference be designated +29° 63138a.

In vol. iii. of the Oxford Astrographic Catalogue (+28°) not only are the components of the double star given separately (Nos. 69921 and 2), but two other fainter stars (Nos. 69920 and 3), closely adjacent.

The places and ϕ mag. of these four stars for 1900.0 may be given as:—

		^h	^m	^s			
+28 69920		12.1	22	27	18.98	28	15 25".17
+28 69921	= 29° 65138	11.3			19.10		55.26
+28 69922	= 29 65138a	11.5			19.34		54.96
+28 69923		12.7			19.59		33.96

The primary star for the 6-min. and 3-min. exposures is visible and measurable on each of the three plates 536, 2567, 2889 taken of this region. On plate 206 (1892.73) the s.f. star is very diffused, faint, and really immeasurable, the second exposure being quite invisible. On plate 536 the primary star has both images good, the s.f. star is decidedly fainter for the first exposure, the second exposure of this star is practically invisible. On plate 2567 (1907.54) both images of both stars are plainly visible, but the relative difference in the intensity of their images on this plate is nearly zero, certainly a much less marked difference than on plates 206 and 536. And on plate 2889 (1911.97) the primary star is distinctly fainter than the s.f. star by 0.2 mag. for the 6-min. exposure, though the second exposure gives the s.f.

star image as slightly less intense than the primary star. The differences are small, and may be partly due to local photographic effects.

The images have been measured and the positions and distances determined for plates 536, 2567, and 2889 as follows:—

536	1894.75	94°7	3°55
2567	1907.54	93°7	3°51
2889	1911.97	92°0	3°72

The centennial Proper Motion deduced from plates 536 and 2889 for $+29^{\circ} 65' 13.8''$ is $4''.7$.

Catalogue of Radiant-Points of Shooting Stars, 1898–1915.

By A. King.

The present paper embodies the principal results of my observations of meteors in the years 1898–1915. The paths were recorded chiefly at Leicester and in North Lincolnshire, but many observations were made in Scotland, Holland, and elsewhere.

In the description of each shower, in many cases more details have been included than is usually done. It seemed to me that this will not only be of assistance to future observers in that very difficult department of the work—the assigning of individual meteors to their correct centres—but may also help to throw some light on the vexed question of stationary radiation.

Some radiants are described as “diffuse.” A diffuse radiant may be caused by one or both of two things—

1. Incorrect observation of individual paths, or errors on the globe or chart used in projection.
2. A cause inherent in the shower itself. The Andromedids and the Quadrantids are well known in this respect.

The names in the last column are, with three exceptions, those of Denning's General Catalogue.

Several of the radiants determined in the early part of the work have been discarded. I had not then had so much experience in the observation of meteors, and on reconsideration found the positions hardly substantiated. Nearly all the early radiants have been reprojected, both on a celestial globe and on a meteor-chart, so that the revised positions sometimes differ from those previously published.

No radiant found from less than five paths has been included unless warranted by the undoubted identity of the meteors and the accuracy of their observation.

In some cases the year of observation is not stated. This indicates that the meteors were recorded in various years.

Position. R.A. Decl.		No. of Mete- ors.	Dates of Observation.	Description.	Name.
3	+25½	9	July 26-Aug. 21, 1903	Very swift. Well-defined max., Aug. 21	α Andromedids *
4	+35	6	July 22-Aug. 14, 1901- 3-4.	Very swift; somewhat long. One headed, with a spark tail. Most active in Aug.	π Andromedids
9	+48	7	July 28-Aug. 14, 1899	Rather swift	α Cassiopeids
18	+59	10	July 31-Aug. 13, 1899	Short; streaks	δ Cassiopeids
25	+62½				
19	+58	5	July 13-Aug. 14, 1904	Swift; shortish. Streaks? White? Radiant very definite	„
26	+59	5	July 24-Aug. 14, 1903	Swift; shortish; reddish. No streaks	„
22	+42	13	July 28-Aug. 14, 1899	Swift; streaks. Somewhat long	γ Andromedids
22	+44	5	July 29-Aug. 12, 1900	Very swift; ruddy. One streaked	„
23	+42	7	July 21-Aug. 15, 1909- 15	Very swift. Mostly Aug. 12-15	„
26	+44	5	Sept. 21, 1901	Swift. Two meteors headed	„
26	+44	6	Aug. 3-12, 1904	Very swift; brightish; streaks	„
24	+50	6	July 26, 1904, and Aug. 9-12, 1901-3	Rather faint; swift	54 Andromedids†
25	+50	6	Aug. 9-11, 1899	Swift; streaks	„
28	+70½	5	July 28-Aug. 21, 1900- 3	Short; swift	50 Cassiopeids
28	+71½	5	July 21, Aug. 11-25, 1904, 1909-11	Quick; very short	„
30	+47	5	July 23-Aug. 11, 1900- 2	Rather swift; bluish; streaks. Rather bright; some headed and trained. Radiant well defined	θ Perseids
33½	+18½	12	Oct. 8-Nov. 20, 1898- 99 and 1906-8.	Slowish; somewhat long. Trains? Greenish; mostly in Oct. Radiant well marked	α Arietids
40	+38½	10	Sept. 27-Oct 8, 1899	Swift; streaks; rather short. Max. Oct. 2	β Perseids
41	+40	6	Oct. 30-Nov. 6, 1899	Rather swift. Streaks? One trained	„

* Denning's " α Andromedids" consist of two showers, one at $3^{\circ}+27^{\circ}$ and the other at $5^{\circ}+20^{\circ}$. It is the first of these, previously observed only by Mr. Denning himself, which is under consideration here.

† I think this centre, intermediate in position betwixt the δ Cassiopeids ($19^{\circ}+58^{\circ}$) and the γ Androms. ($24^{\circ}+43^{\circ}$), is distinct from either. The star 54 Andromedids is almost at radiant, hence name given here. In 1901-4 the radiant was precise and the meteors well observed.

No.	Position.		No. of Mete- ors.	Dates of Observation.	Description.	Name.
	R. A.	Decl.				
21	41 $\frac{1}{2}$	+39 $^{\circ}$	4	Aug. 3 and 11, 1904	Pale blue; small, pear-shaped heads; trains with central bright points; swift. One streaked. Meteors not bright. Radiant very exact. Max. Aug. 11	β Perseids
22	43	+21	9	July 13-Aug. 21, 1899-1907	Longish paths; heads; trains; streaks. Very swift. Radiant diffuse	ϵ Arietids
	α .	δ .				
23	39 $^{\circ}$	+4	4	Sept. 27-Nov. 6, 1899	Moderate, greenish	α Cetids
24	44	+56	7	Sept. 6-21, 1901	Very swift	η Perseids

The Great Persid Shower.

25	25	+52	4	July 27, 1910-11	Swift; streaks	Perseids
26	28	+53	4	July 27-28, 1902	" "	"
27	34	+55 $\frac{1}{2}$	6	Aug. 3, 1903	Veryswift; streaks	"
28	35 $\frac{1}{2}$	+56 $\frac{1}{2}$	5	" 3, 1911	" " brightish	"
29	36	+57	7	" 4, 1903	" " "	"
30	38	+56 $\frac{1}{2}$	10	" 6, 1904	" " "	"
31	42	+57	4	" 9, 1902	Swift; streaks	"
32	44	+57	11	" 9, 1899	" " "	"
33	44	+58	9	" 10, 1902	Very swift; streaks	"
34	44	+57	7	" 10, 1909	Swift; streaks	"
35	44	+56 $\frac{1}{2}$	6	" 10, 1915	" " "	"
36	45	+57	29	" 10, 1899	Veryswift; streaks	"
37	46 $\frac{1}{2}$	+57	14	" 11, 1901	" " "	"
38	45 $\frac{1}{2}$	+57 $\frac{1}{2}$	32	" 11, 1904	" " bright	"
39	45	+56 $\frac{1}{2}$	17	" 11, 1909	" " "	"
40	45	+59	7	" 11, 1914	" " brightish	"
41	46	+57	20	" 11, 1899	" " "	"
42	48	+58	12	" 12, 1900	" " bright	"
43	47	+58	13	" 12, 1901	" " rather bright	"
44	48	+56 $\frac{1}{2}$	6	" 12, 1902	" " "	"
45	46 $\frac{1}{2}$	+57 $\frac{1}{2}$	13	" 12, 1904	" " bright	"
46	47	+58	6	" 12, 1909	Swift; streaks	"
47	46	+56 $\frac{1}{2}$	9	" 12, 1914	Veryswift; streaks. Mostly bright	"
48	48	+58 $\frac{1}{2}$	14	" 12, 1915	Swift; streaks	"
49	48	+58	7	" 12, 1899	Very swift; streaks; bright	"
50	47	+58 $\frac{1}{2}$	7	" 13, 1914	Bright; swift; streaks	"

No.	Position.		No. of Mete- ors.	Dates of Observation.	Description.	Name.
	α .	δ .				
51	48	+58 $\frac{1}{2}$	7	„ 13, 1915	Swift ; streaks	Perseids
52	49	+57	6	„ 13, 1899	„ „	„
53	48	+59 $\frac{1}{2}$	6	„ 14, 1903	„ „ brightish	„
54	50 $\frac{1}{2}$	+58 $\frac{1}{2}$	4	„ 14, 1904	Very swift ; streaks	„
55	48	+58	5	„ 14, 1914	Swift ; streaks	„
56	50	+59	3	„ 14, 1899	„ „ definite	„
57	51	+58	4	„ 15, 1901	„ „ bright	„
58	45	+64	5	Aug. 8-11, 1899	Swift ; streaks. Bright. Trains?	P. III. 94. Camelids
59	50	+20 $\frac{1}{2}$	4	Oct. 30-Nov. 6, 1899	Bright ; slow	ζ Arietids
60	58	+8	8	Oct. 28-Dec. 16	Starlike ; slowish. Mostly long, bright and trained. Radiant well marked	ϵ Taurids
61	60 $\frac{1}{2}$	+59 $\frac{1}{2}$	7	Aug. 8-14, 1904	Swift ; streaks. Some bright. Radiant de- finite	P. III. 208. Camelids
62	61	+58	12	July 28-Aug. 14, 1899	Max. Aug. 8-14. Very swift ; brightish ; streaks. Many were headed	„
63	61	+59	5	Aug. 4-21, 1900	Max. Aug. 21, swift	„
64	61	+59	7	Aug. 10-13, 1909-15	Swift ; streaks. Several headed. Two trained, in which cases no streaks. Mostly yellow. Radiant definite	„
65	61	+48 $\frac{1}{2}$	6	Aug. 11, 12, 1901	Very swift ; streaks. Two were headed	μ Perseids
66	61	+47 $\frac{1}{2}$	7	Aug. 10-19, 1903-15	Very swift ; streaks. Brightish. One trained. ? White. Radiant de- finite	„
67	70	+17 $\frac{1}{2}$	4	Dec. 1, 3, 1902	Short ; rather swift. Radi- ant well defined	α Orionids
68	73	+53	13	July 29, 30, and Aug. 9-13, 1899	Swift ; streaks. Three headed, one trained. Well-defined max. Aug. 12	8 Camelids
69	75	+42	5	Aug. 4-21, 1903	Swift ; streaks	α Aurigids
70	75	+41	7	Aug. 15 - Sept. 21, 1900-1	Swift ; streaks. Mostly headed. Radiant diffuse	„
71	75	+41	9	Aug. 9-Sept. 25	Very swift ; streaks. Rather bright. Mostly headed and white	„
72	91	16	4	Oct. 18, 1900-1	Swift ; streaks. Rather bright	Orionids
73	91 $\frac{1}{2}$	+15	4	Oct. 22, 1898	Very swift ; streaks	„
74	109	+31	4	Dec. 8-13, 1899-1900	Short ; rather swift	Geminids

No.	Position.		No. of Mete- ors.	Dates of Observation.	Description.	Name.
	α .	δ .				
75	111	+33 $\frac{1}{2}$	6	Dec. 13, 1906	Short; swift	Geminids
76	121	+32	5	Dec. 7-13, 1908, 1909, 1915	Slowish; some trained. Rather bright. Mostly greenish and starlike	β Geminids
77	130	+20	4	Nov. 8, etc., 1898, 1899	Long paths. Moderate	δ Cancriids
78	148 $\frac{1}{2}$	+22	10	Nov. 15, 1903	Very swift; bright; streaks	Leonids
79	150	+23	5	Nov. 14, 15, 1903	Swift; bright; streaks *	„
80	150 $\frac{1}{2}$	+21	8	Nov. 15, 1904	Bright; swift; streaks. Two were headed, long- ish and trained	„
81	151	+23	7	Nov. 8-17, 1900-15	Long paths. Very swift; brightish; streaks	„
82	151 $\frac{1}{2}$	+22	7	Nov. 15, 1899	Very swift; streaks. Bright	„
83	151 $\frac{1}{2}$	+22	3	Nov. 15, 1900	Very swift; paths rather long. Radiant well de- fined	„
84	216	-26	5	Apr. 11-24, 1903	Rather swift; bright; long †	Hydrids
85	228	+52	8	Jan. 3, 1903	Moderate; trained; long	Quadrantids
86	232	+53	8	Jan. 3, 1910	Brightish. Moderate; long. Streaks. One trained. Radiant diffuse	„
87	236 $\frac{1}{2}$	+51 $\frac{1}{2}$	4	Apr. 19, 1903	Short; rather swift. Radi- ant. Well defined	τ Herculids?
88	256 $\frac{1}{2}$	+37	6	Apr. 19-22, 1903	Swift. Well defined max.; Apr. 22	π Herculids
89	266	+26	7	Apr. 20-21, 1900	Faint; rather swift	97 Herculids
90	271	+77 $\frac{1}{2}$	4	July 26-Aug. 21, 1900- 3	Slowish; mostly faint; greenish?	ψ Draconids
91	271 $\frac{1}{2}$	+33	12	Apr. 21-22, 1903	Swift; streaks. Many were headed	Lyrids
92	274	+33	6	Apr. 20-21, 1900	Rather swift; streaks. Whitish	„
93	283	+63	6	July 27-Aug. 10, 1902	Rather short; very swift	δ Draconids
94	286	-12	7	July 13-Aug. 16	Slowish; bright; stellar	λ Aquilids
95	299	+54	4	Aug. 7-15, 1904	Very swift. Radiant well defined	33 Cygnids
96	299	+56	12	July (end)-Aug. (beg.) 1911	Very swift. Radiant dif- fuse, but centre well marked	„

* A reprojection of some of the best paths recorded during the great shower of 1903.

† This radiant, which has shown activity only in recent years, was first determined by Professor Herschel, from paths recorded by various observers in 1900 April 16-21, at 218°-31° (*M.N.*, Feb. 1901, p. 247). The position here given is the first individually determined.

No.	Position.		No. of Mete- ors.	Dates of Observation.	Description.	Name.
	α .	δ .				
97	300	- 11	6	July 8-Aug. 15, 1902- 11	Bright; slowish; some trained. Greenish. Paths rather long. Mostly in July. Radiant well de- fined	α Capricornids
98	306	- 17	3	July 24, 1900	Green; moderate; bright. Longish. Radiant well defined	„
99	314	+ 71	5	July 27-Aug. 16, 1911 and 1914	Short even when a long way from radiant; slowish; starlike. Radiant diffuse	ϵ Draconids
100	318	- 9	10	July 28-Aug. 19	Slowish; rather long. Nearly entirely in August. Radiant diffuse, and ex- act place doubtful	β Aquarids
101	324	+ 10	12	July 24-Aug. 12, 1899- 1903	Very swift. White	ϵ Pegasids
102	330	+ 35	4	Mar. 29-Apr. 24, 1903	Slowish. Radiant well de- fined	π Pegasids
103	331	- 12	5	July 27-30, 1911	Longish paths. No trains or streaks. Moderate. Stellar	ι Aquarids
104	337	- 13½	5	July 27-Aug. 4, 1902- 3	Long. Rather swift	δ Aquarids
105	344	+ 46	9	July 28-Aug. 10, 1899	Rather swift; short; white; starlike; streaks	σ Andromedids
106	345	+ 1	6	Aug. 1-Oct. 2, 1899	Slowish. Mostly "thin"*	β Piscids
107	346	+ 1	6	July 30-Aug. 22, 1900	Slowish; mostly greenish and "thin." One trained	„
108	346	+ 1	8	July 28-Aug. 21, 1901- 10	Moderate or rather swift	„
109	346	+ 36	10	Aug.-Sept. 1899	Swift; white?	ι Andromedids
110	351	+ 53	4	July 22-Aug. 21, 1903	Slowish; short; faint	λ Andromedids
111	353	+ 27	6	July 24-28, Aug. 22, 1900	Swift. Radiant well defined	78 Pegasids
112	355	+ 77	10	July 27-Aug. 21, 1899- 1903	Very swift. Radiant well marked. Exactly on γ Cephei	γ Cepheids
113	357	+ 66	5	July 27-Aug. 18, 1899, 1902, 1915	Short; quick (flashing or "snappy"). White or bluish	σ Cepheids
114	358	+ 61	6	July 16-Aug. 12, 1900- 1903	Slowish; "thin"	β Cassiopeids
115	359	+ 59	5	July 28-Aug. 14, 1899	Slowish; "thin"	„
116	359	+ 59	6	Aug. 14-Sept. 21	Moderate	„
117	359	+ 1	5	July 26-Aug. 4, 1903	Rather swift; rather long	44 Piscids

* This refers to such bodies as have little or no apparent diameter, and unless carefully watched produce the appearance, due to persistence of vision, of thin lines across the sky. The term "stellar" might be used, but is liable to confusion with "starlike."

Notes on the Perseids.

Dates of Maximum.—The following are the dates of the maximum as determined in various years. In the missing years the materials were too scanty to indicate the epoch of maximum, owing usually to the sky having been obscured for several nights on and about the date:—

- 1899 Aug. 10.
- 1901 Aug. 11 or 12, probably 12.
- 1902 Aug. 11–12, probably Aug. 12.
- 1903 Aug. 12?
- 1904 Aug. 11 (early morning of 12th).
- 1907, probably Aug. 11.
- 1909 Aug. 11. Well marked. Observed in Holland.
- 1911, probably Aug. 12. Observed in Holland.
- 1914 Aug. 12, but only slightly stronger than on 11th.
- 1915 Aug. 12. Well marked.

The *strength* of the shower varies, too, year by year. In 1898 the display was decidedly strong; in 1899 fairly so. 1901 and 1902 brought returns of average strength. In 1904 the shower was again rather strong, while in 1907 it was not of normal activity. In 1909 the shower was pretty normal, but perhaps slightly weak. 1914 and 1915 produced weakish displays especially in 1915 (the latter as observed in Scotland).

In deducing these facts, due allowance has been made for the prevalence of bright moonlight and mist or cloud at about the time of maximum. It must not be forgotten, though, that difference of locality of observation may make a great difference in the observed strength of a given shower, since watchers widely separated see different sets of meteors.

Many of the Perseids are headed, and some trained, the trains generally having a central row of bright points, decreasing in size from head to tip. The shower usually furnishes a brighter display at and about the times of maxima.

The table below sets forth a comparison betwixt the observed places and those of an ephemeris of the shifting radiant given by Mr. W. F. Denning in 1901 (*Monthly Notices of the R.A.S.*, vol. lxii. p. 169). In working out the average positions, day by day, the radiants obtained in 1899 have been advanced a day, *i.e.* the position found on 1899 Aug. 9 has been averaged with those of Aug. 10 in other years, and so on. This is necessary owing to the lapse of a day in 1900.

Date.	Av. Position (A.K.)		No. of Radiants averaged.	Position (Ephemeris, W.F.D.).	
July 27	25°	+52°	1	27°1	+53°2
„ 27-28	28	+53	1	27°7	+53°4
Aug. 3	34·8	+56	2	35°1	+55°3
„ 4	36	+57	1	36°4	+55°5
„ 6	38	+56·5	1	38°9	+56°0
„ 9	42	+57	1	42°9	+56°7
„ 10	44	+57·2	4	44°3	+56°9
„ 11	45·4	+57·4	5	45°7	+57°1
„ 12	47·1	+57·5	8	47°1	+57°3
„ 13	47·7	+58·3	3	48°5	+57°5
„ 14	48·9	+58·3	4	50°0	+57°7
„ 15	50·5	+58·5	2	51°4	+57°8

Mr. Denning remarks, in the place referred to above, that it is “quite possible that the radiant should be placed a little farther north, particularly at about the period of the maximum.” The mean declinations given in the second column above would certainly seem to bear out this statement.

Scunthorpe, N. Lincs. :
1916 April 3.

